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Robust Optimization of the Self-scheduling and Market Involvement for an Electricity Producer

Lima, R. M.

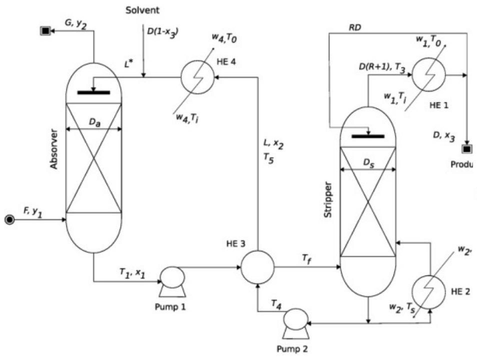
2006 - PhD

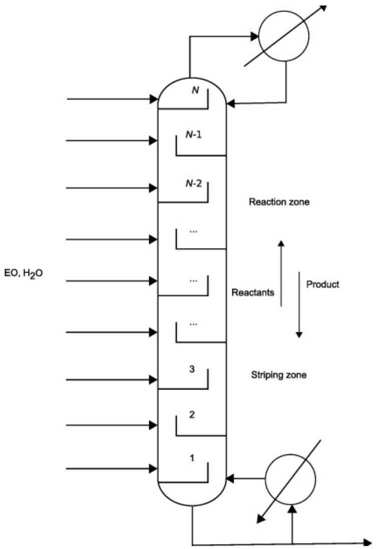
Department of Chemical Engineering

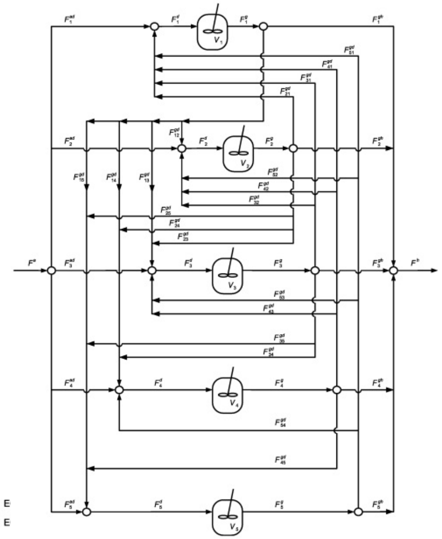
Faculty of Engineering, University of Porto, Portugal

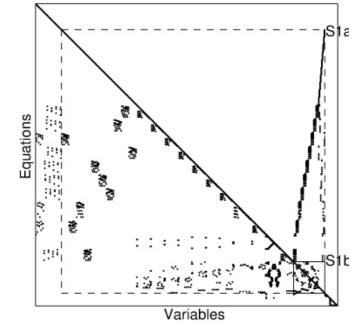
An integrated strategy for simulation and optimization of chemical processes

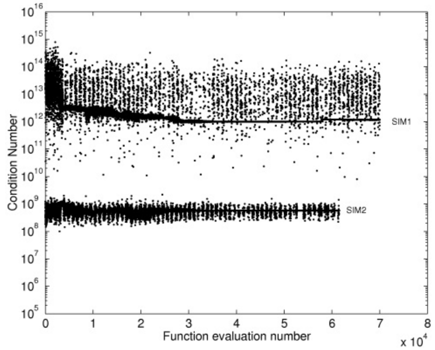
Salcedo, R. L. and Barbosa, D.











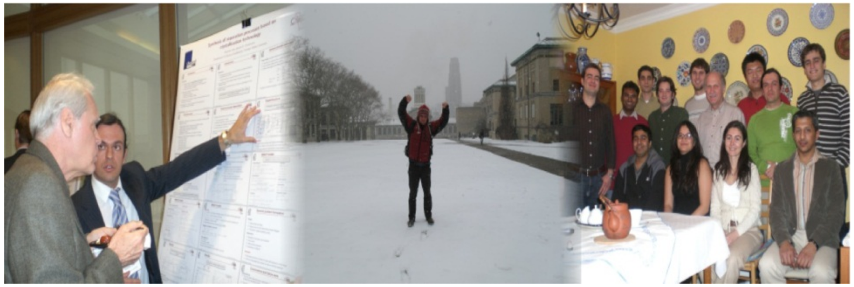
Lima, R.M.

2006-2008 – Post-doc

Ignacio E. Grossmann

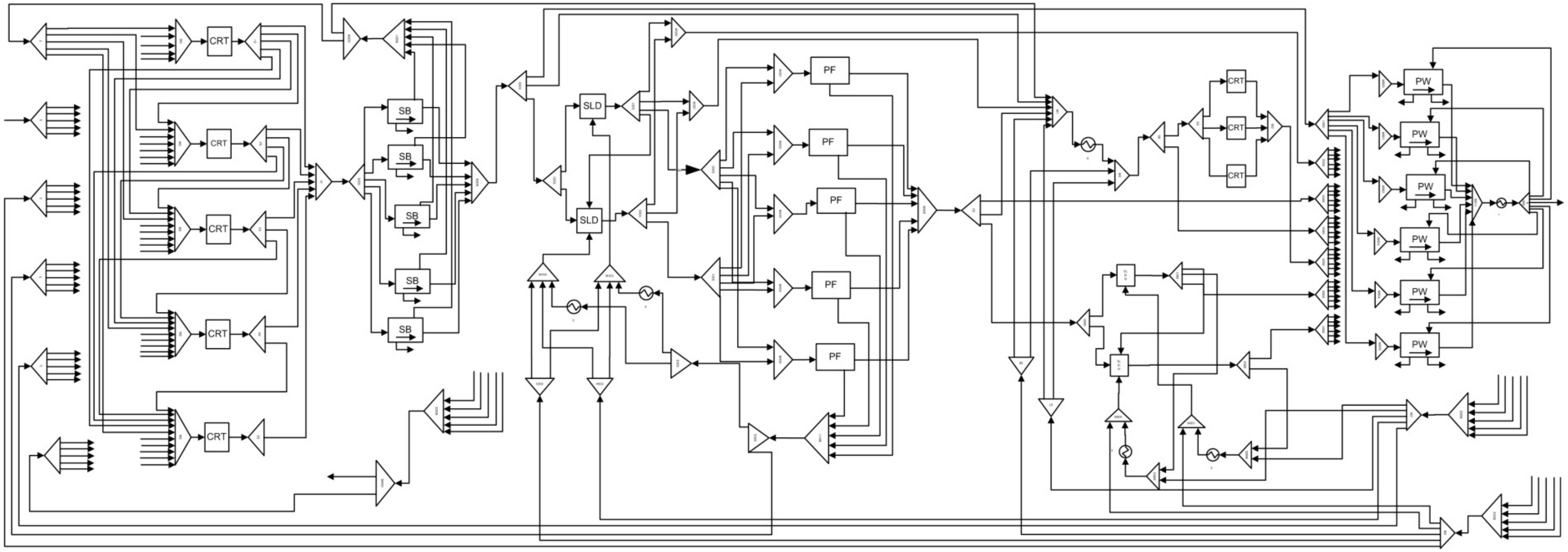
Carnegie Mellon University, PA, USA

Department of Chemical Engineering



Optimal synthesis of p-xylene separation processes based on crystallization

Process synthesis, complex MINLP problems



Lima, R.M.

2008 – 2011 – Researcher

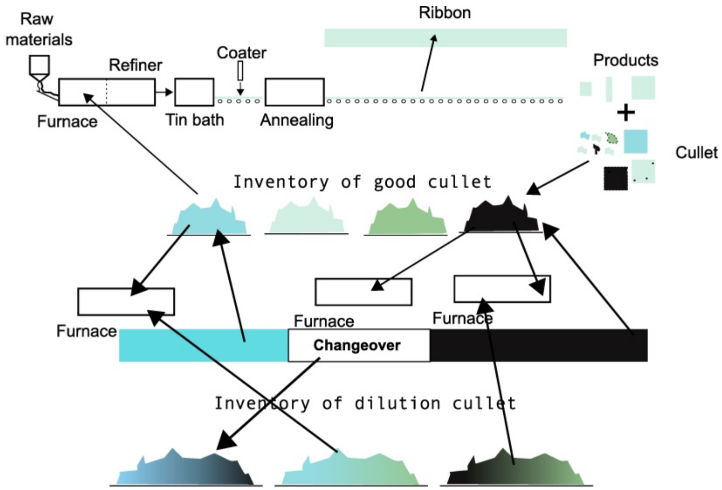
Ignacio E. Grossmann

PPG Industries

Carnegie Mellon University, PA, USA

Department of Chemical Engineering

Planning and long-term scheduling of single stage multi-product continuous lines with a complex recycling structure



Lima, R. M.

2011 – Co-Fund Marie Curie Fellowship

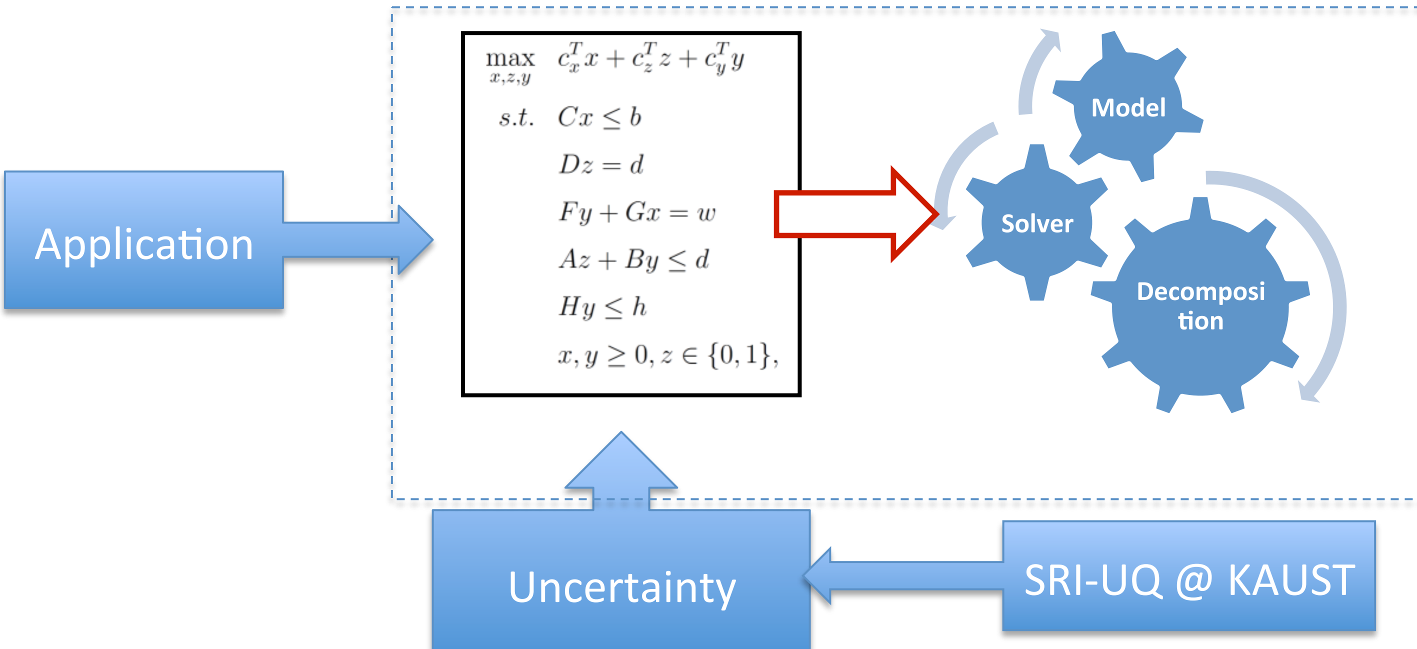
Laboratório Nacional de Energia e Geologia, I.P. (LNEG), Lisbon, Portugal

Project: Planning and Scheduling of Optimal Mix of Renewable Sources in Sustainable Power Systems

- Optimization models and solution methods
- Interdisciplinary work

Lima@KAUST

- Research Scientist, joined SRI-UQ@KAUST on October, 2014
- Working with Omar Knio and Ibrahim Hoteit
- Optimization under uncertainty
- Merge Uncertainty Quantification with Optimization
- Focus on high impact applications



Motivation and objectives

- **Research developments and challenges**
  - Developments in two stage adaptive Robust Optimization (RO)
    - Bertsimas and Slim, 2003
    - Bertsimas et al., 2011
    - Bertsimas et al., 2013
    - Thiele et al., 2009

Adaptive Robust Optimization for the Security Constrained Unit Commitment Problem

Dimitris Bertsimas, *Member, IEEE*, Eugene Litvinov, *Senior Member, IEEE*, Xu Andy Sun, *Member, IEEE*, Jinye Zhao, *Member, IEEE*, and Tongxin Zheng, *Senior Member, IEEE*

- **Problem features**
  - Complementarity of energy sources: hydro and wind
  - Uncertainty due to renewable energy sources
  - Deregulation of electricity markets
    - Scheduling problems to minimize operational costs
    - Maximize profit by their interaction with the electricity market
      - Electricity pool

Develop an optimization framework based on RO to support the decision making of electricity producers in a market environment.

RO Overview

- Aims to find a robust solution for a problem under uncertainty
  – Where by robust it is meant that such solution is the optimal for the worst conditions within an uncertainty set describing the uncertainty

- RO advantages
  1. Under specific conditions leads to computational tractable problems
  2. Results can be very reliable, since worst case situations are considered
  3. It does not require a distribution of probabilities

- RO disadvantages

  0.

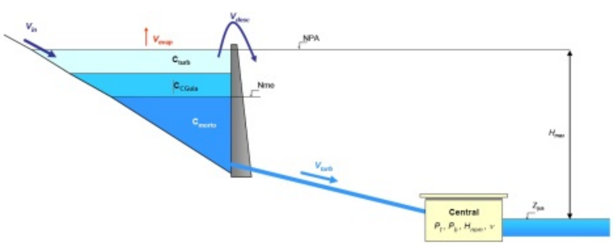
  1. **Crude** representation of the uncertainty
  2. Solutions can be very conservative

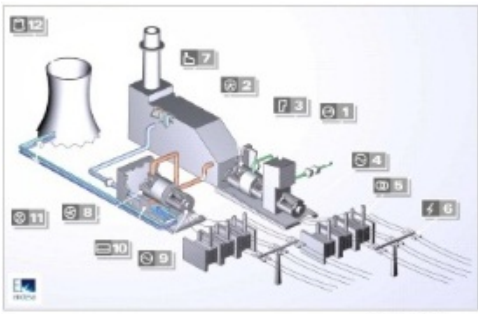
Meaningful uncertainty sets for RO -> Big Data available

Control the conservatism level

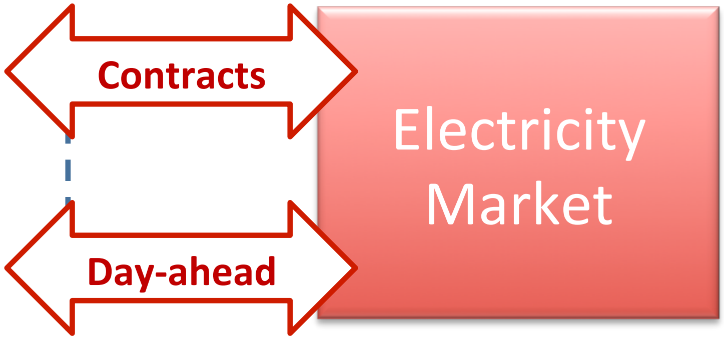
Problem definition

- Mixed power generation system operating in an electricity market





Fonte: Endesa



- Combinatorial scheduling problem
- Constraints on the technical operation of the units

Decision framework

2^{nd} stage decisions

Thermal plant dispatch

Power output levels subject to commitment

Pump-storage hydro plant dispatch

Power output levels

Power consumption to pump

Sell and buy electricity in the pool

1 week

Resolution: 1 hour

Self-scheduling

Fix commitment of the thermal unit → Fix 0-1 variables – on/off status of thermal unit

Forward contracting

Sign selling or buying contracts → Decide buy or sell, power and price

1^{st} stage decisions

Problem statement

- **Given**
  - Electricity producer with a portfolio of generation units
    - Operating constraints of the units
    - The system can be operated as a virtual power system
  - The producer can interact with the market
    - Buy or sell through forward contracts and the pool
  - The time horizon of 1 week, with the resolution of 1 hour
  - Forward contracts format
  - Electricity price forecasts and error limits
  - Wind power forecast and error limits
- **Determine**
  - Power generation schedule by unit
  - Hourly electricity sold and bought in the pool, and by contracts
- **Maximize**
  - **Operational profit**

2-stage adaptive RO framework

Deterministic model

$$
\begin{aligned}
\max_{x,z,y} \quad & c_x^T x + c_z^T z + c_y^T y \\
s.t. \quad & Cx \leq b \\
& Dz = d \\
& Fy + Gx = w \\
& Az + By \leq d \\
& Hy \leq h \\
& x, y \geq 0, z \in \{0,1\},
\end{aligned}
$$

Multi-period MILP problem,
x, *y* continuous variables
z binary variables

R … fit

realization of the stochastic parameters

within W

Uncertainty on wind power and electricity prices

2-stage adaptive RO

$$
\begin{aligned}
\max_{x,z} \quad & c_x^T x + c_z^T z + R(x,z) \\
s.t. \quad & Cx \leq b \\
& Dz = d \\
& x \geq 0, z \in \{0,1\},
\end{aligned}
$$

$$
\begin{aligned}
R(x,z) = \min_{w, c_y} \quad \max_{y} \quad & c_y^T y \\
s.t. \quad & Fy = w - Gx \\
& By \leq d - Az \\
& Hy \leq h \\
& y \geq 0 \\
s.t. \quad w, c_y \in W. &
\end{aligned}
$$

Comparison of 2-Stage Formulations

2-stage adaptive RO

$$
\begin{aligned}
\max_{x,z} \quad & c_x^T x + c_z^T z + R(x,z) \\
s.t. \quad & Cx \leq b \\
& Dz = d \\
& x \geq 0, z \in \{0,1\},
\end{aligned}
$$

$$
\begin{aligned}
R(x,z) = \min_{w,c_y} \quad \max_{y} \quad & c_y^T y \\
s.t. \quad & Fy = w - Gx \\
& By \leq d - Az \\
& Hy \leq h \\
& y \geq 0 \\
s.t. \quad w, c_y \in W. &
\end{aligned}
$$

2-stage Stochastic Programming

$$
\begin{aligned}
\max_{x,z} \quad & c_x^T x + c_z^T z + R(x,z) \\
s.t. \quad & Cx \leq b \\
& Dz = d \\
& x \geq 0, z \in \{0,1\},
\end{aligned}
$$

$$
R(x,z) = \mathbb{E}_\xi Q(x, \xi(w))
$$

$$
\begin{aligned}
Q(x, \xi(w)) = \quad \max_{y} \quad & c_y(w)^T y \\
s.t. \quad & Fy = h(w) - G(w)x \\
& By(w) \leq d(w) - A(w)z \\
& Hy(w) \leq g(w) \\
& y(w) \geq 0
\end{aligned}
$$

2-stage adaptive RO framework (cont.)

Recourse problem

$$
\begin{aligned}
R(x,z) = \min_{w,c_y} \quad & \max_{y} \quad c_y^T y \\
& s.t. \quad Fy = w - Gx \\
& \qquad\quad By \le d - Az \\
& \qquad\quad Hy \le h \\
& \qquad\quad y \ge 0 \\
s.t. \quad & w, c_y \in W.
\end{aligned}
$$

Inner of the recourse problem

$$
\begin{aligned}
IR(x,z,w,c_y) = \max_{y} \quad & c_y^T y \\
s.t. \quad & Fy = w - Gx \\
& By \le d - Az \\
& Hy \le h \\
& y \ge 0,
\end{aligned}
$$

Convex LP problem

Assuming strong duality, the dual of IR is given by

$$
\begin{aligned}
DIR(x,z,w,c_y) = \min_{\alpha,\beta,\mu} \quad & (w - Gx)^T \alpha + (d - Az)^T \beta + h^T \mu \\
s.t. \quad & F^T \alpha + B^T \beta + H^T \mu \ge c_y \\
& \alpha \in \mathbb{R},\ \beta \ge 0,\ \mu \ge 0
\end{aligned}
$$

Next step: Merge the outer problem of the Recourse with the Dual DIR

2-stage adaptive RO framework (cont.)

Reformulated recourse problem

$$
\begin{aligned}
LDR(x,z) = \min_{w,c_y,\alpha,\beta,\mu} \quad & (w - Gx)^T \alpha + (d - Az)^T \beta + h^T \mu \\
s.t. \quad & F^T \alpha + B^T \beta + H^T \mu \geq c_y \\
& \alpha \in \mathbb{R},\ \beta \geq 0,\ \mu \geq 0 \\
& w, c_y \in W.
\end{aligned}
$$

2-stage adaptive RO

$$
\begin{aligned}
\max_{x,z} \quad & c_x^T x + c_z^T z \quad + \min_{w,c_y,\alpha,\beta,\mu} \quad (w - Gx)^T \alpha + (d - Az)^T \beta + h^T \mu \\
& \qquad\qquad\qquad s.t. \quad F^T \alpha + B^T \beta + H^T \mu \geq c_y \\
& \qquad\qquad\qquad\qquad \alpha \in \mathbb{R},\ \beta \geq 0,\ \mu \geq 0 \\
& \qquad\qquad\qquad\qquad w, c_y \in W \\
s.t. \quad & Cx \leq b \\
& Dz = d \\
& x \geq 0, z \in \{0,1\},
\end{aligned}
$$

This is **a nontrivial optimization problem** because of the **bi-level** structure
Difficult to solve with a standard solver

(Dual) Constraint Generation Algorithm

(Thiele et al., 2009; Zhang and Guan, 2009; Jiang et al. 2010; Zugno and Conejo, 2013)

{Initialization}
$LB := -\infty,\ UB := +\infty,\ k := 1$
$x^k := x^0,\ z^k := z^0$
$O := \emptyset$
while $(UB - LB)/LB \leq \varepsilon$ **do**
{Solve subproblem}

$$
\begin{aligned}
LDR(x^k, z^k) = \min_{w, c_y, \alpha, \beta, \mu} \quad & \left(w - Gx^k\right)^T \alpha + \left(d - Az^k\right)^T \beta + h^T \mu \\
s.t. \quad & F^T \alpha + B^T \beta + H^T \mu \geq c_y \\
& \alpha \in \mathbb{R},\ \beta \geq 0,\ \mu \geq 0 \\
& w, c_y \in W.
\end{aligned}
$$

$LB := \max\{LB, c_x^T x^k + c_z^T z^k + LDR(x^k, z^k)\}$
$O := O \cup \{k\}$
{Solve Master problem}

$$
\begin{aligned}
PF(x, z) = \max_{x, z, \Theta} \quad & c_x^T x + c_z^T z + \Theta \\
s.t. \quad & \Theta \leq \left(w^k - Gx\right)^T \alpha^k + (d - Az)^T \beta^k + h^T \mu^k, \quad k \in O \\
& Cx \leq b \\
& Dz = d \\
& x \geq 0, z \in \{0, 1\}, \Theta \in \mathbb{R}
\end{aligned}
$$

$UB := \min\{UB, PF(x, z)\}$
$k := k + 1$
end while

Primal Constraint Generation Algorithm

Master Problem

$$
\begin{aligned}
PF(x,z) = \max_{x,z,\Theta} \quad & c_x^T x + c_z^T z + \Theta \\
s.t. \quad & \Theta \le \left(w^k - Gx\right)^T \alpha^k + (d - Az)^T \beta^k + h^T \mu^k, \quad k \\
& Cx \le b \\
& Dz = d \\
& x \ge 0, z \in \{0,1\}, \Theta \in \Re
\end{aligned}
$$

Recourse Problem

$$
\begin{aligned}
R(x,z) = \min_{w,c_y} \quad \max_{y} \quad & c_y^T y \\
s.t. \quad & Fy = w - Gx \\
& By \le d - Az \\
& Hy \le h \\
& y \ge 0 \\
s.t. \quad & w, c_y \in W.
\end{aligned}
$$

Introduce a copy of the primal variables ***y***

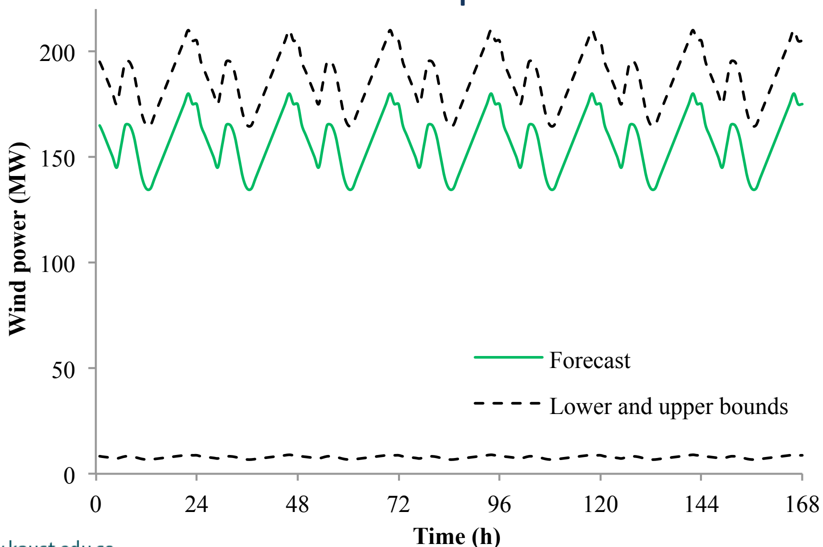
$$
\begin{aligned}
PF(x,z) = \max_{x,z,\Theta} \quad & c_x^T x + c_z^T z + \Theta \\
s.t. \quad & \Theta \le \left(w^k - Gx\right)^T \alpha^k + (d - Az)^T \beta^k + h^T \mu^k, \quad k \in O \\
& \Theta \le c_y^{Tk} y^k, \quad k \in O \\
& Fy^k = w^k - Gx, \quad k \in O \\
& By^k \le d - Az, \quad k \in O \\
& Hy^k \le h, \quad k \in O \\
& Cx \le b \\
& Dz = d \\
& x \ge 0, z \in \{0,1\}, y^k \ge 0, \Theta \in \mathbb{R},
\end{aligned}
$$

Uncertain Polyhedral Sets

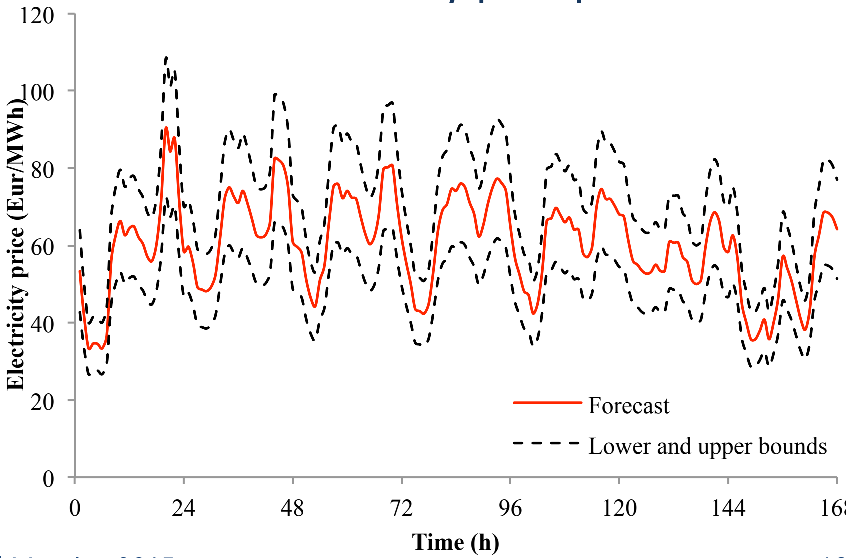
- Uncertainty is described by polyhedral sets: built around a nominal value
  - Forecast value
  - Forecast error -> lower and an upper bound
- This is an alternative approach to a scenario framework built from a probability distribution

$$w_t = \overline{w}_t + z_t^+ w_t^u - z_t^- w_t^l$$

Wind power



Electricity pool prices



Risk management

- The solution is at one of the extreme points of the convex set
  - May lead to over conservative solutions
- In RO risk management is implemented by budget constraints

$$\sum_t z_t^+ + z_t^- \leq \Gamma$$

Γ – Budget parameter
z_t^+, z_t^- - 0-1 variables

High Γ – high number of periods exhibit deviations from $\overline{w}_t$ → **Conservative approach**

Low Γ – low number of periods exhibit deviations from $\overline{w}_t$ → **Risk prone approach**

Wind power uncertainty set

$$W^w = \left\{ w_t \geq 0, z_t^+, z_t^- \in \{0,1\}, \forall t, : w_t = \overline{w}_t + z_t^+ w_t^u - z_t^- w_t^l, \sum_t z_t^+ + z_t^- \leq \Gamma \right\}$$

Electricity pool prices uncertainty set

$$W^\lambda = \left\{ \lambda_t \geq 0, y_t^+, y_t^- \in \{0,1\}, \forall t, \lambda_t = \overline{\lambda}_t + y_t^+ \lambda_t^u - y_t^- \lambda_t^l, \sum_t y_t^+ + y_t^- \leq \Gamma \right\}$$

Characterization of the subproblem

$$DIR(x,z,w,c_y) = \min_{\alpha,\beta,\mu} \quad (w - Gx)^T \alpha + (d - Az)^T \beta + h^T \mu$$
$$s.t. \quad F^T\alpha + B^T\beta + H^T\mu \geq c_y$$
$$\alpha \in \mathbb{R},\ \beta \geq 0,\ \mu \geq 0$$

$$LDR(x,z) = \min_{w,c_y,\alpha,\beta,\mu} \quad (w - Gx)^T \alpha + (d - Az)^T \beta + h^T \mu$$
$$s.t. \quad F^T\alpha + B^T\beta + H^T\mu \geq c_y$$
$$\alpha \in \mathbb{R},\ \beta \geq 0,\ \mu \geq 0$$
$$w, c_y \in W.$$

$$\begin{aligned}
\text{Profit}_{\text{pool}} \quad = \quad \min \Bigg\{ & \sum_t \Bigg\{ \Bigg[\sum_f \sum_j \left(f_{f,j}^{sell} - f_{f,j}^{buy} \right) - w_t \Bigg] \alpha_t \Bigg\} \\
+ & \sum_{i \in TH} \sum_t \left[\left(P_i^u u_{i,t} \right) \beta_{i,t} + \left(-P_i^l u_{i,t} \right) \gamma_{i,t} \right] + \sum_{i \in TH} \sum_{t=1} \left[\left(P0_i + RU_i U0_i + SU_i u_{i,t}^{up} \right) \zeta_{i,t} \right] \\
+ & \sum_{i \in TH} \sum_{t>1} \left[\left(RU_i u_{i,t-1} + SU_i u_{i,t}^{up} \right) \eta_{i,t} + \left(RD_i u_{i,t} + SD_i u_{i,t}^{dn} \right) \vartheta_{i,t} \right] \\
+ & \sum_{i \in HY} \sum_{t=1} \left[\left(V0_i + GQ_i^{in} \right) \mu_{i,t} \right] + \sum_{i \in HY} \sum_{t>1} \left(GQ_i^{in} \nu_{i,t} \right) \\
+ & \sum_{i \in HY} \sum_t \left(Q_i^u \varpi_{i,t} + Q_i^u \rho_{i,t} - V_i^l \tau_{i,t} + V_i^u \upsilon_{i,t} \right) - \sum_{i \in HY} \sum_{t=tf} \left(V_i^E \varphi_{i,t} \right) \Bigg\}
\end{aligned}$$

Characterization of the subproblem (cont.)

$$LDR(x,z) = \min_{w,c_y,\alpha,\beta,\mu} \quad (w-Gx)^T\alpha + (d-Az)^T\beta + h^T\mu$$
$$s.t. \quad F^T\alpha + B^T\beta + H^T\mu \geq c_y$$
$$\alpha \in \mathbb{R},\ \beta \geq 0,\ \mu \geq 0$$
$$w, c_y \in W.$$

α_t – dual variable

λ_t – electricity price in the pool

$$-\alpha_t \geq \lambda_t \quad \forall t \qquad :p_t^{sell}$$
$$\alpha_t \geq -\lambda_t \quad \forall t \qquad :p_t^{buy}$$

$$\sum_{i\in TH} p_{i,t} + \sum_{i\in HY} ptb_{i,t} + p_t^{buy} + \sum_f \sum_j f_{f,j}^{buy} + w_t = \sum_{i\in HY} pp_{i,t} + p_t^{sell} + \sum_f \sum_j f_{f,j}^{sell}, \quad \forall t,$$

$$\nu_{i,t} - \nu_{i,t+1} - \tau_{i,t} + \upsilon_{i,t} \geq 0 \quad \forall i \in HY, t > 1, t < tf \qquad :v$$
$$\nu_{i,t} - \varphi_{i,t} - \tau_{i,t} + \upsilon_{i,t} \geq 0 \quad \forall i \in HY, t = tf \qquad :v$$
$$G\mu_{i,t} - K_i^t H_i \xi_{i,t} + \varpi_{i,t} \geq 0 \quad \forall i \in HY, t = 1 \qquad :q$$
$$G\nu_{i,t} - K_i^t H_i \xi_{i,t} + \varpi_{i,t} \geq 0 \quad \forall i \in HY, t > 1 \qquad :q$$
$$\alpha_t + \xi_{i,t} \geq 0 \quad \forall i \in HY, t \qquad :ptb$$
$$-\alpha_t + \pi_{i,t} \geq 0 \quad \forall i \in HY, t \qquad :pp$$
$$-G\mu_{i,t} - K_i^p H_i \pi_{i,t} + \rho_{i,t} \geq 0 \quad \forall i \in HY, t = 1 \qquad :qp$$
$$-G\nu_{i,t} - K_i^p H_i \pi_{i,t} + \rho_{i,t} \geq 0 \quad \forall i \in HY, t > 1 \qquad :qp$$

Linearization of the subproblem

Linearization of $w_t\alpha_t$

Definition of w_t

$$w_t = \overline{w}_t + z_t^+ w_t^u - z_t^- w_t^l$$

$$\alpha_t \geq \overline{\lambda} + y_t^+ \lambda^u - y_t^- \lambda^l \quad \forall t$$

$$w_t\alpha_t = \overline{w}_t\alpha_t + z_t^+ w_t^u \alpha_t - z_t^- w_t^l \alpha_t, \quad \forall t,$$

$$\alpha_t \geq -\left(\lambda_t + y_t^+ \lambda_t^u - y_t^- \lambda_t^l\right), \quad \forall t,$$

Substitution

$$v_t^+ \leq -\left(\overline{\lambda}_t - \lambda_t^l\right) z_t^+, \quad \forall t,$$

$$v_t^+ \leq \alpha_t + \left(\overline{\lambda}_t + \lambda_t^u\right)\left(1 - z_t^+\right), \quad \forall t,$$

Linearization

$$v_t^- \geq -\left(\overline{\lambda}_t + \lambda_t^u\right) z_t^-, \quad \forall t,$$

$$v_t^- \geq \alpha_t, \quad \forall t,$$

$$v_t^- \leq M_3 z_t^-, \quad \forall t,$$

$$v_t^- \geq \alpha_t - M_4\left(1 - z_t^-\right), \quad \forall t,$$

Based on

$$-\alpha_t \geq \lambda_t \quad \forall t$$

$$\alpha_t \geq -\lambda_t \quad \forall t$$

$$\lambda_t = \overline{\lambda}_t + y_t^+ \lambda_t^u - y_t^- \lambda_t^l,$$

Remarks

1. Master and Sub-Problem are MILP problems.
2. The Sub-Problem is always bounded for any first stage decisions (complete recourse) if the option to buy energy from the pool is considered.
3. If the MILP Sub-Problem is not solved to optimality then
   I. The LB is not computed with the best solution of the Sub-Problem found, but with the best MILP LB, $\overline{LDR}(x^k, z^k)$

   $$LB := \max\{LB, c_x^T x^k + c_z^T z^k + \overline{LDR}(x^k, z^k)\}$$

   II. The integer solution obtained is still a valid bound

   $$\Theta \leq LDR(x^k, z^k) \leq \underline{LDR}(x^k, z^k)$$

Results

- **Computational experiments**
  - **Case 1**
    - **1 thermal unit named G1**
    - 1 pumped-storage hydro unit
    - 1 wind farm
  - **Case 2**
    - **1 thermal unit named G2**
    - 1 pumped-storage hydro unit
    - 1 wind farm
  - 2 Algorithms: Dual and Primal
  - 3 Instances of electricity prices: EP1, EP2, EP3
  - Risk management: 5 values for the budget parameter
- Models implemented in GAMS, on a computer with an Intel Core i7@3.07GHz CPU, 64 bits, and 8Gb of RAM. The MILP problems are solved with CPLEX 12.5.

Case 1 – Computational results

Maximum CPU time set to 1500s and 0.1% gap.

EP	Γ	Dual constraint generation			
		P (€)	Gap (%)	# Iter	CPU^1(s)
EP1	0	4,936,002	0.00	1	1
EP1	10	4,713,789	0.00	1	2
EP1	100	3,463,714	0.06	1	8
EP1	150	3,035,343	0.00	2	4
EP1	168	2,935,271	0.00	1	1

onstraint generation			
(%)	# Iter	CPU^2(s)	ΔCPU (%)
0.00	1	2	-50
0.00	1	2	0
0.06	1	8	0
0.00	2	5	-20
0.00	1	2	-50
0.00	1	2	-50
0.00	1	5	0
0.03	1	15	0
0.05	1	4	0
0.00	1	2	-50
0.00	1	2	100
0.01	1	2	200
0.01	6	18	-17
0.03	8	33	-9
0.03	8	30	-10

$= (CPU^2 - CPU^1)/CPU^1$

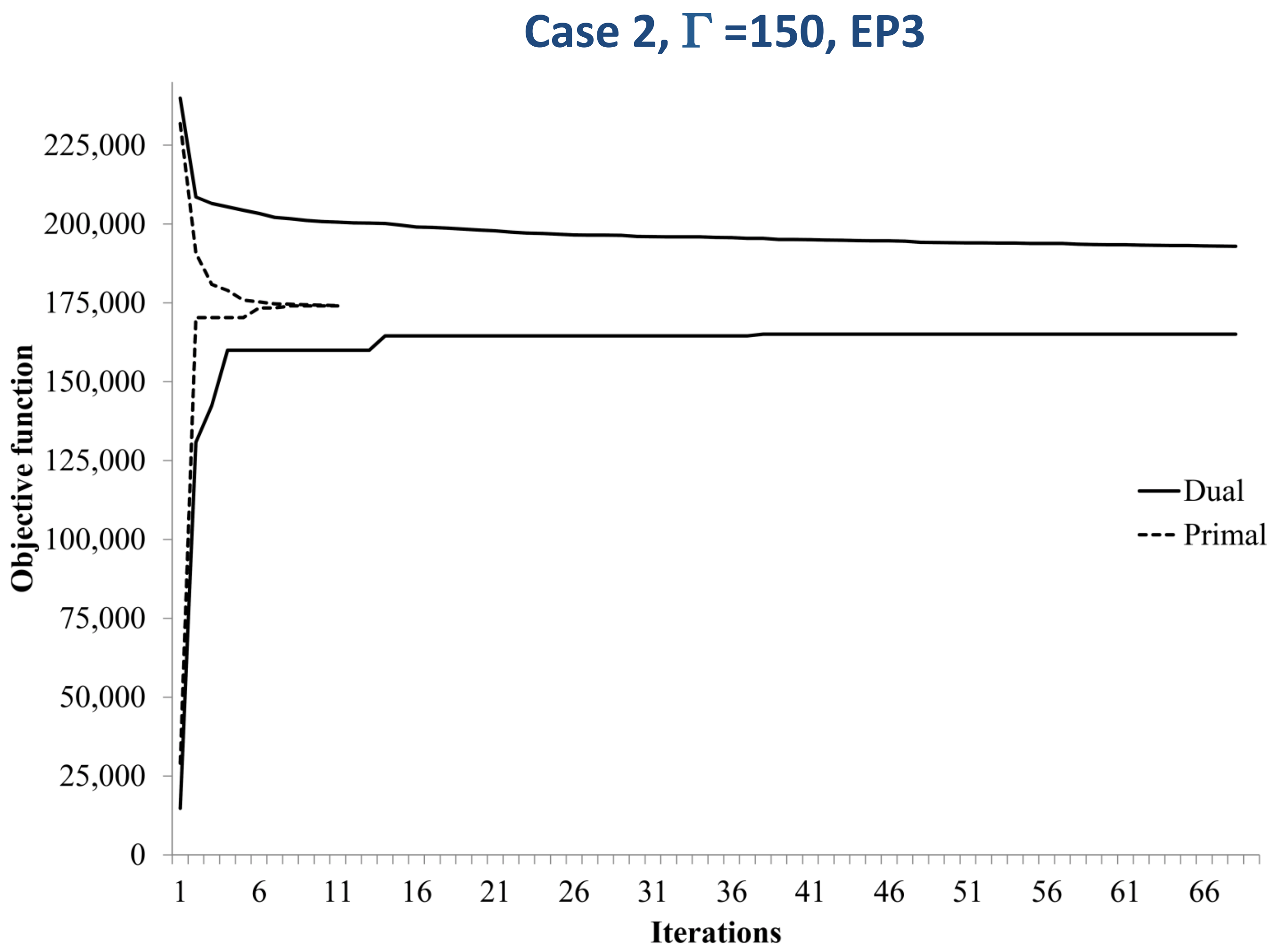
Case 2 – Computational results

EP	Γ	Dual constraint generation			
		P (€)	Gap (%)	# Iter	CPU¹(s)
EP1	0	1,935,466	0.09	16	20
EP1	10	1,773,240	0.09	17	22
EP1	100	831,836	3.70	31	1,534
EP1	150	465,544	5.90	31	1,506
EP1	168	372,917	1.99	31	1,532
EP2	0	1,530,169	0.09	26	32
EP2	10	1,412,875	0.10	26	259
EP2	100	623,943	6.99	30	1,538
EP2	150	328,213	7.32	34	1,533
EP2	168	247,803	5.13	31	1,535
EP3	0	805,971	0.93	166	1,507
EP3	10	692,294	1.19	148	1,528
EP3	100	217,334	12.76	74	1,508
EP3	150	165,068	16.90	68	1,523
EP3	168	165,868	16.24	65	1,556

. constraint generation			
·ap (%)	# Iter	CPU²(s)	ΔCPU (%)
0.00	1	2	900
0.00	1	2	1,000
0.96	26	1,518	-
0.91	28	1,564	-
1.12	29	1,538	-
0.00	1	2	1,500
0.00	1	24	979
2.04	25	1,532	-
1.55	28	1,524	-
1.99	28	1,551	-
0.00	1	2	-
0.00	2	4	-
0.05	10	162	-
0.09	11	298	-
0.09	12	394	-

·U = (CPU² - CPU¹)/CPU¹.

Convergence profiles: Dual vs Primal

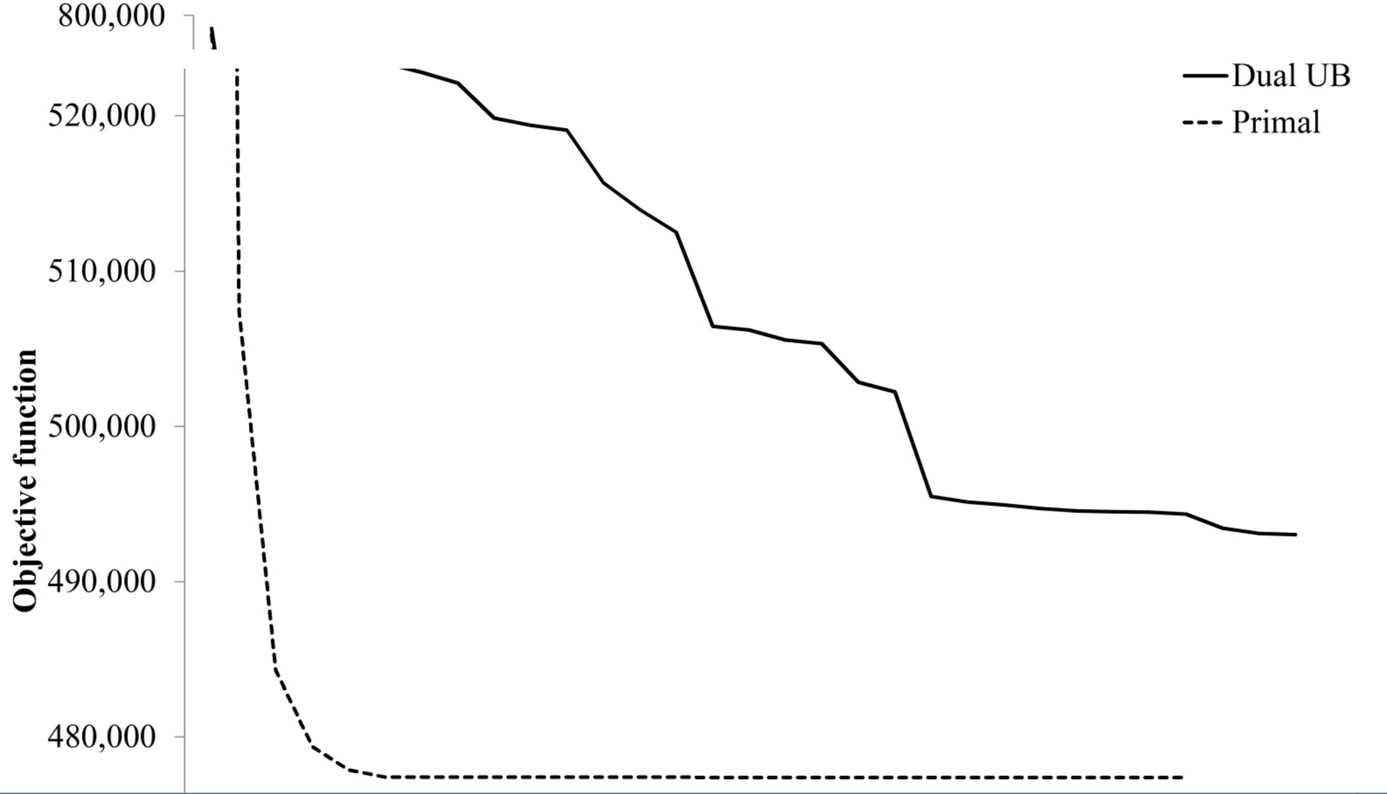


16.90% gap

Convergence profiles: Dual vs Primal

Case 2, Γ =150, EP1

Both algorithms do not converge

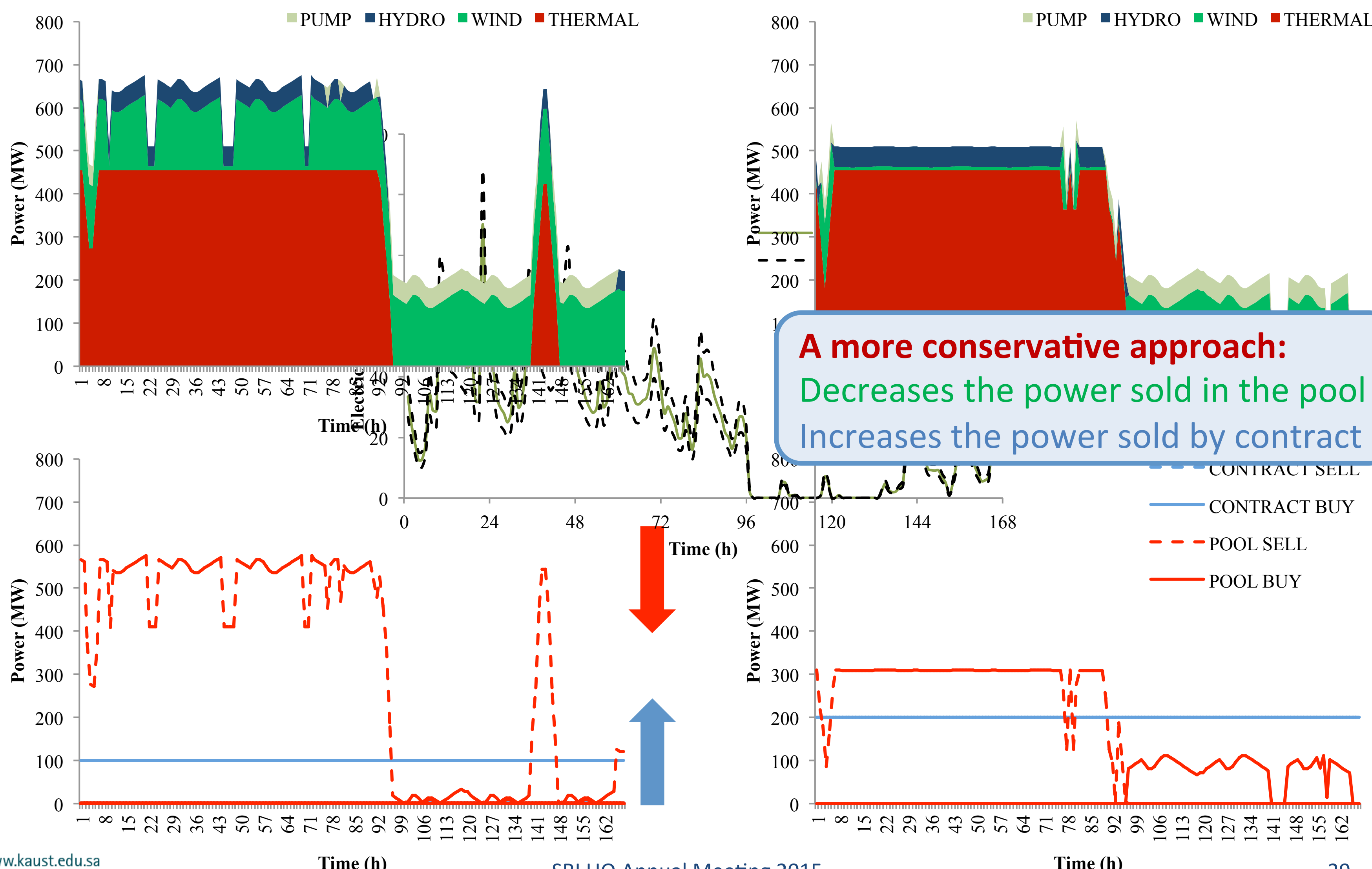


The Primal Constraint Generation Algorithm cannot close the gap
MILP Sub-Problem is not solved to optimality

Case 1, EP3 – Scheduling and Market Results

Γ = 10, Risk prone approach

Γ = 100, Conservative approach



A more conservative approach:

Decreases the power sold in the pool

Increases the power sold by contract

Risk management results: budget parameter

Case 1

Γ	EP1 FC (MW)	EP1 P^{sell} (MWh)	EP1 P^{buy} (MWh)
0	0	103,756	0
10	100	85,275	0
100	300	37,848	0
150	300	30,875	0
168	300	28,286	0

More conservative approaches:
Decreases the power sold in the pool
Increases the power sold by contract

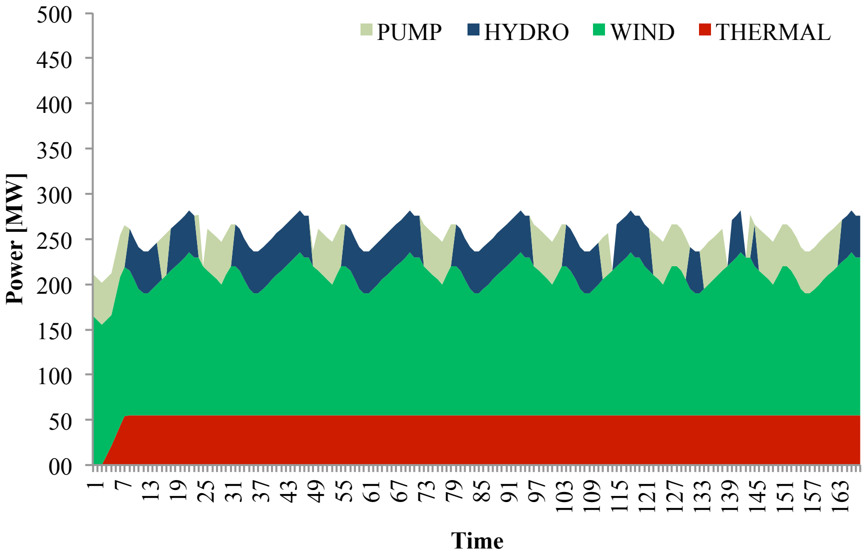
Case 2

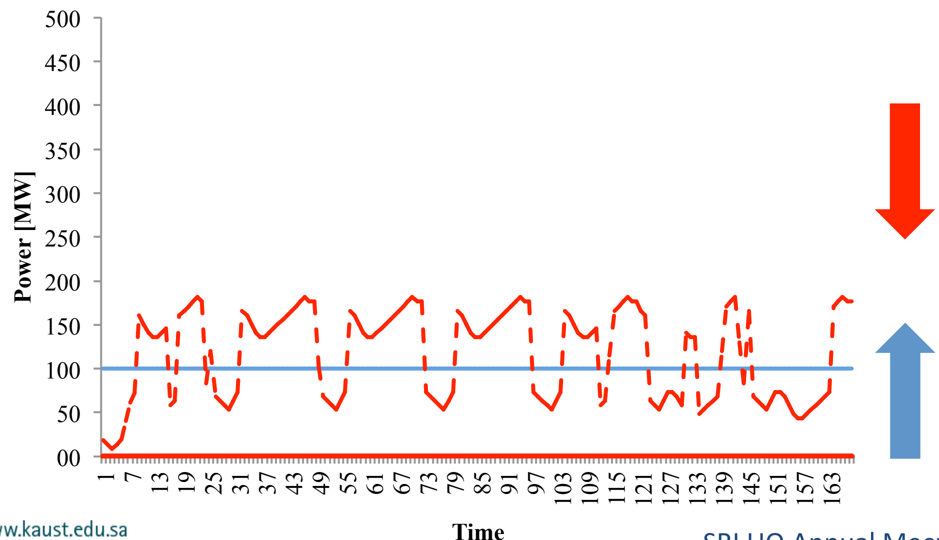
Γ	EP1 FC (MW)	EP1 P^{sell} (MWh)	EP1 P^{buy} (MWh)
0	0	36,276	0
10	100	17,809	0
100	107	3,395	553
150	83	3,533	3,678
168	82	2,434	4,857

More conservative approaches:
Decreases the total energy

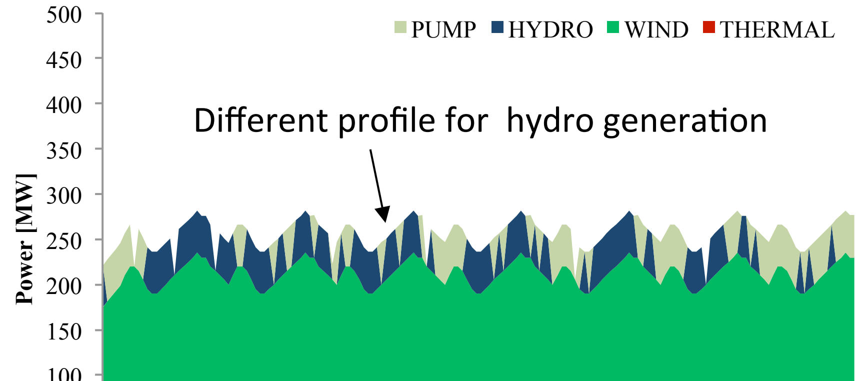
Case 2, EP1 – Perfect information for Wind

Γ = 10, Risk prone approach





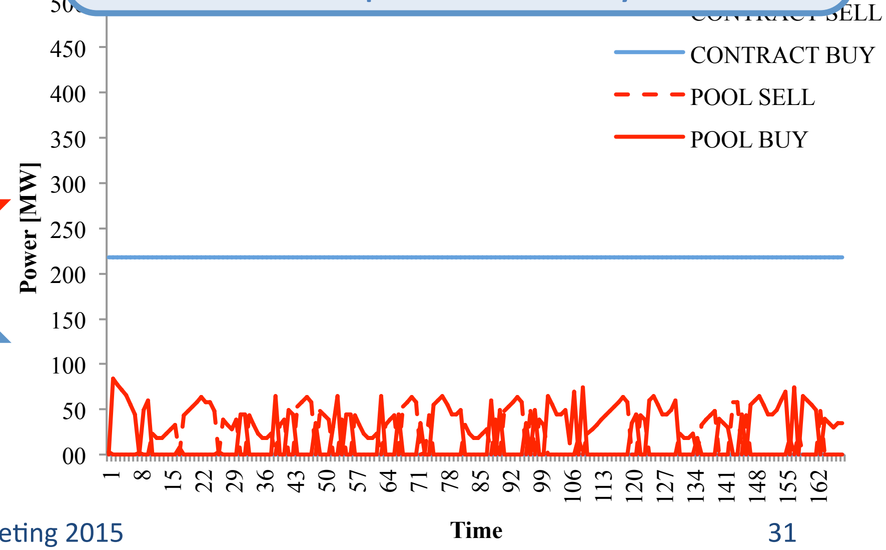
Γ = 168, Conservative approach



A more conservative approach:

Decreases the power sold in the pool

Increases the power sold by contract



Risk management results: budget parameter

Case 1

	EP1		
Γ	FC (MW)	P^{sell} (MWh)	P^{buy} (MWh)
0	0	103,756	0
10	100	86,956	0
100	300	53,356	0
150	300	53,356	0
168	300	53,356	0

More conservative approaches:
Decreases the power sold in the pool
Increases the power sold by contract

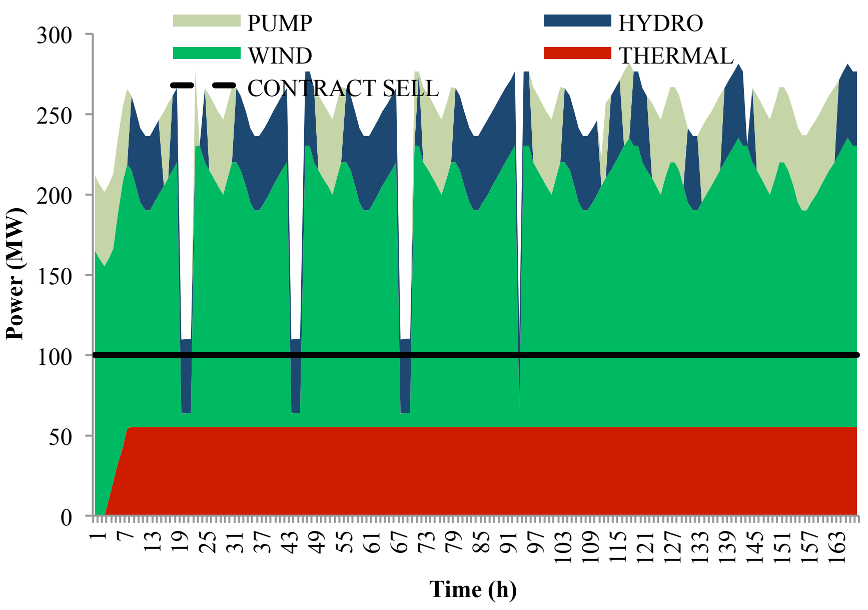
Case 2

	EP1		
Γ	FC (MW)	P^{sell} (MWh)	P^{buy} (MWh)
0	0	36,276	0
10	100	19,476	0
100	213	4,078	3,443
150	218	3,705	3,885
168	218	3,528	3,707

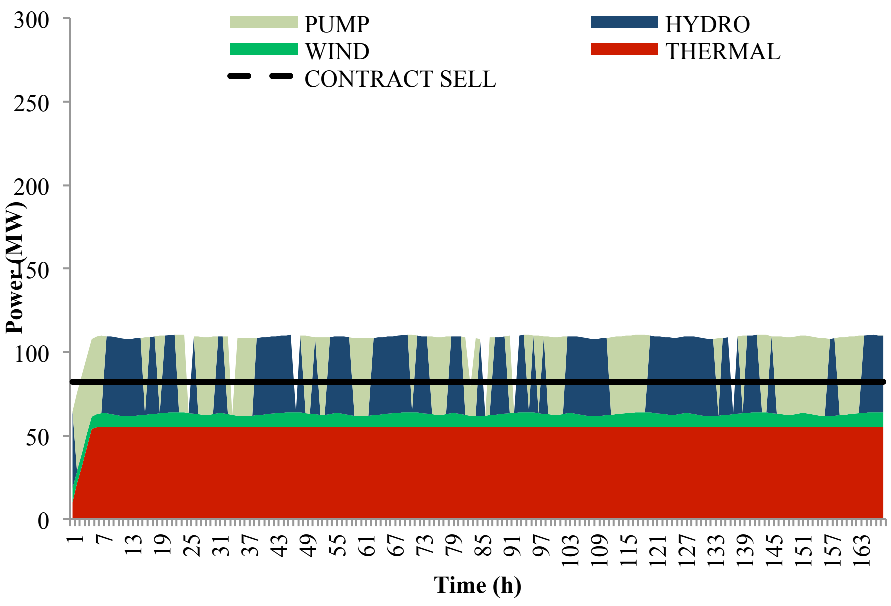
More conservative approaches:
Decreases the power sold in the pool
Increases the power sold by contract

Case 2 - Budget parameter vs contract selection

Γ = 10, Risk prone approach



Γ = 168, Conservative approach



Conclusions and final remarks

- Robust optimization framework
  1. The Sub-Problem has **full recourse** as long as the producer has the **option to buy electricity**, this simplifies the algorithm.

  2. The **two variants** of the constraint generation algorithm have a **similar performance** with exception for some cases where the **Primal version is better**.

  3. Some **MILP** Sub-Problems are **not solved to optimality**
     I. The constraint generation algorithm **does not close the gap**
     II. The convergence profile **seems** to indicate that it has obtained the optimal solution

Conclusions and final remarks (cont.)

- Risk management
  1. Uncertainty only in electricity prices
     - More conservative approaches lead to lower profits (as expected)
     - Selection of forward contracts to hedge against the volatility of the pool
  2. Uncertainty on electricity prices and wind power
     - More conservative approaches lead to lower profits (as expected)
     - It is difficult to foresee and isolate the relation between the conservatism level and the contract selection and pool involvement

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