Time Frequency Array signal processing: A synergistic relationship between Time frequency methods and Sensor array processing

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Outline

Array signal processing

- Signal processing extracts information from measured signals
- Array signal processing uses a group of sensors:
  - Signal enhancement / noise reduction
    - Coherence adding
    - Spatial filtering
  - Source / Channel characterizations:
    - Number of sources
    - Location "direction finding"
    - Waveforms "information from the sources"

Applications

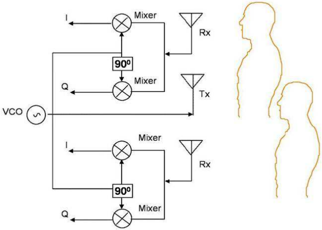
- Wireless communications
- Interference mitigation
- Radar / Sonar
- Biomedical
- Speech
- Seismic
- ...

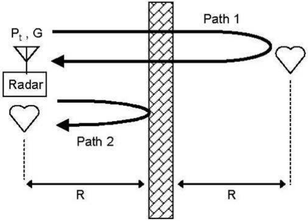
Example: Passive Radar

- GSM-based passive radar from Nanyang Technological University, Singapore

Example: Through the Wall sensing

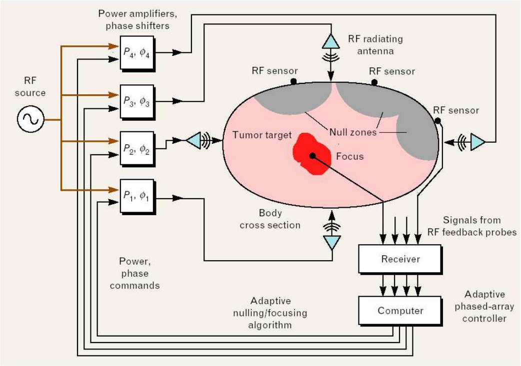
- Doppler heart sensing radar (Dung Phuong Nguyen et al. 2007)
- Radar operator's cardiopulmonary activity may be detected due to reflection of the radar signal at the wall.
- Array processing allows separation of the heart-motion signals of the radar operator and the subject behind a wall





Example: Adaptive Phased-Array Hyperthermia Treatment of Cancer

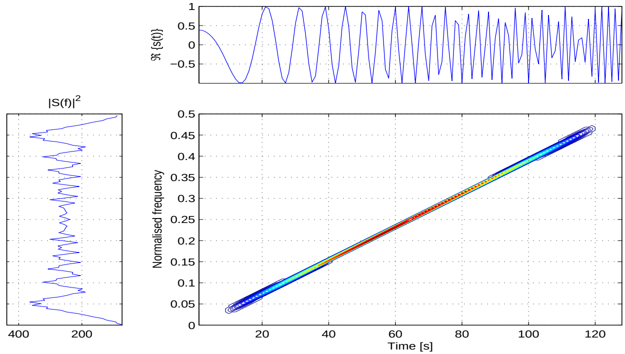
- Minimally invasive system of coherent electromagnetic radiating antennas used to heat a deep-seated malignant tumor (Alan J. Fenn et al.)



Time frequency concept

- Non stationary signals: frequency content varies in time
- Time-frequency distributions (TFDs) represent signal energy versus time and frequency simultaneously
- Effective tools for analyzing non stationary signals

Linear Frequency-Modulated (LFM) signal



Spectrogram

- A well-known TF representation is the short-time Fourier transform (STFT):

$$S_h(t,f) = \int_{-\infty}^{+\infty} s(\tau)h(\tau - t)e^{-j2\pi f\tau}d\tau$$

- Spectrogram

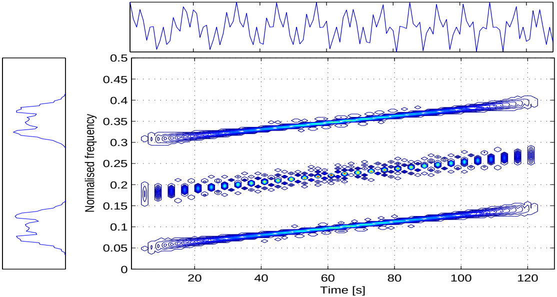
$$\rho^{SPEC}(t,f) = |S_h(t,f)|^2 = |\int_{-\infty}^{+\infty} s(\tau)h(\tau - t)e^{-j2\pi f\tau}d\tau|^2$$

Wigner-Ville Distribution

$$W_z(t,f) \triangleq \int_{-\infty}^{+\infty} z(t+\tau/2)z^*(t-\tau/2)e^{-j2\pi f\tau}d\tau = \underset{\tau\to f}{\mathcal{F}}\{z(t+\frac{\tau}{2})z^*(t-\frac{\tau}{2})\}$$

- Advantages: high TF resolution
- Disadvantage: suffer from cross-terms for multiple component signal.

WVD of the sum of two LFM (chirp) signals



General Class of Quadratic Time Frequency Distributions

$$D_z(t,f) = \underset{\tau \rightarrow f}{\mathcal{F}} \{ G(t,\tau) \underset{t}{*} z(t+\frac{\tau}{2}) z^*(t-\frac{\tau}{2}) \}$$

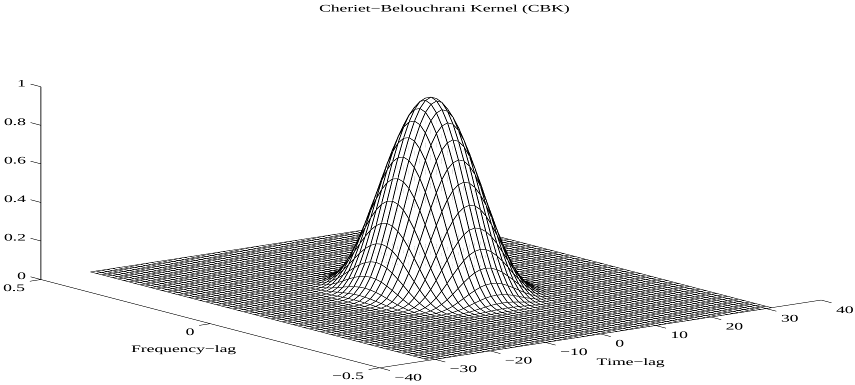
$G(t,\tau)$: time-lag kernel
$g(\nu,\tau) = \mathcal{F}_{t \rightarrow \nu}\{G(t,\tau)\}$: Doppler-lag filter kernel.

- The filter kernel determines the TFD and its properties.

Reduced Interference Distributions: Some common filter kernels

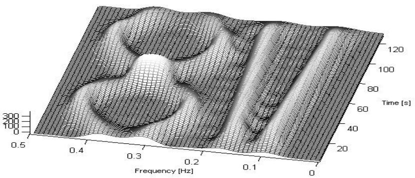
Name	**"Doppler-lag" Kernel**: $g(\nu,\tau)$
WVD	1
SPEC	$\int_{-\infty}^{+\infty} h(u+\frac{\tau}{2})h^*(u-\frac{\tau}{2})e^{-j2\pi\nu u}du$
CWD	$e^{-\nu^2\tau^2/\sigma}$
BJD	$\frac{\sin(2\pi\alpha\nu\tau)}{2\pi\alpha\nu\tau}$
B: "*Barkat-Boashash*"	$\lvert\tau\rvert^{\beta}\frac{2^{2\beta-1}}{\Gamma(2\beta)}\lvert\Gamma(\beta+j\pi\nu)\rvert^2$
CB: "*Cheriet-Belouchrani*"	$\begin{cases} e^{\frac{1}{2}(\frac{\gamma}{\nu^2+\tau^2-1}+\gamma)} & \text{for } \nu^2+\tau^2<1 \\ 0 & \text{otherwise} \end{cases}$

Cheriet-Belouchrani Kernel

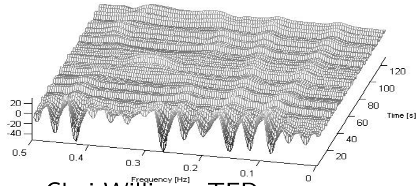


M. Cheriet and A. Belouchrani, "Method and System for Measuring the Energy of a Signal," World Intellectual Property Organization, Patent number WO02088760 A2, Nov. 2002.

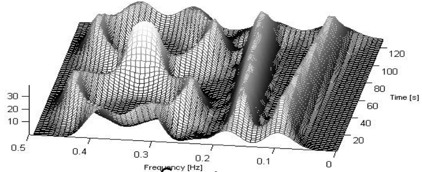
Sum of two sinusoidal FM signals and two linear chirp signals



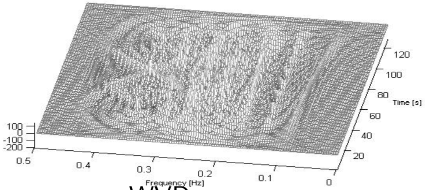
Cheriet-Belouchrani



Choi-Williams TFD



Spectrogram



WVD

M.Abed, A.Belouchrani, M.Cheriet and B.Boashash, Time-Frequency Distributions based on Compact Support Kernels: Properties and Performance Evaluation, IEEE Trans. on Signal processing, June 2012.

Non stationarity versus stationarity

- Conventional array signal processing assumes stationary signals
- When non stationary signals, the achievable performance are reduced
- Stationarity hypothesis was motivated by the need of estimating sample statistics through temporal averaging
- Instead of being a shortcoming, non stationarity would better be a source of information
- Consider non stationarity for the design of efficient algorithms

Non stationarity versus stationarity

- Question: How can we exploit the non stationarity in array signal processing ?
- One can use the Spatial Time Frequency Distributions (STFDs)
- STFDs are the generalization of the time frequency distributions from scalar signals to a vector of multi-sensor signals.

The Spatial Time Frequency Distributions (STFD)

- Given an analytic vector signal $\mathbf{z}(t)$, the Spatial TFDs (STFDs) are defined as

$$\mathbf{D}_{\mathbf{zz}}(t, f) = \underset{\tau \rightarrow f}{\mathcal{F}} \{ G(t, \tau) \underset{t}{*} \mathbf{z}(t + \frac{\tau}{2}) \mathbf{z}^*(t - \frac{\tau}{2}) \},$$

$G(t, \tau)$: time-lag kernel

- STFD of a vector signal is a matrix whose diagonal entries are the auto-TFDs of the vector components and the off-diagonal entries are the cross-TFDs

Structure under linear model

- Linear model of a vector signal $\mathbf{z}(n)$:

$$\mathbf{z}(n) = \mathbf{A}\mathbf{s}(n),$$

$\mathbf{A}$: $K \times L$ matrix ($K \geq L$)
$\mathbf{s}(n)$: $L \times 1$ vector (the source/desired) signal vector.

- STFDs structure under linear model:

$$\mathbf{D}_{\mathbf{zz}}(n,k) = \mathbf{A}\mathbf{D}_{\mathbf{ss}}(n,k)\mathbf{A}^H$$

$\mathbf{D}_{\mathbf{ss}}(n,k)$: source STFD of vector $\mathbf{s}(n)$
The STFD $\mathbf{D}_{\mathbf{ss}}(n,k)$ is diagonal (off-diagonal) for each source auto-term (cross-term) points

Selection of auto-terms and cross-terms

- Intuitive procedure: Consider t-f points corresponding to the maximum energy in the time frequency plane
- This has shown some limitations in practical situations
- An alternative solution: exploits existence of only one source at some auto-term point
- At such points, each auto-term STFD matrix is of rank one, or at least has one "large" eigenvalue
- One can use rank selection criteria to select auto-term points as those corresponding to rank one STFD matrices

Selection of auto-terms and cross-terms

- For example:

$$\text{if } \left| \frac{\lambda_{\max}\{\hat{\mathbf{D}}_{\mathbf{zz}}(n,k)\}}{\text{norm}\{\hat{\mathbf{D}}_{\mathbf{zz}}(n,k)\}} - 1 \right| > \epsilon$$

$$\longrightarrow \text{ decide that } (n,k) \text{ is a auto-term}$$

  typically, $\epsilon = 1E-4$
  $\lambda_{\max}\{\hat{\mathbf{D}}_{\mathbf{zz}}(n,k)\}$: the largest eigenvalue of $\hat{\mathbf{D}}_{\mathbf{zz}}(n,k)$
- Others efficient procedures for the selection of the signal auto terms can be found in recent literature

Selection of auto-terms and cross-terms

- The selection of auto-terms in the time frequency domain is still an open problem

Success of any STFD based technique depends on the performance of the auto-term selection procedure

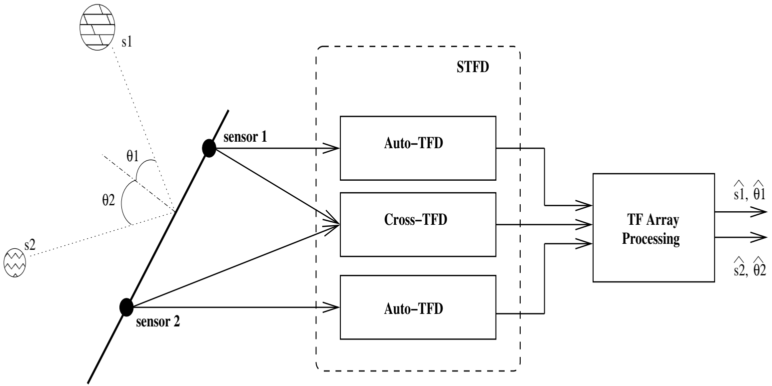
STFDs Applications

- STFDs structures permit the application of subspace techniques to solve a large class of problems as
  - Channel estimation
  - Equalization
  - Blind source separation "Blind Spatial filtering"
  - High-resolution Direction Of Arrival (DOA) estimation

STFDs Applications

- In the blind Spatial filtering problem, the STFDs allow the extraction of Gaussian sources with identical spectral shape but with different time frequency signatures
- STFDs also allow to extract "more sources than sensors"
- In the area of DOA finding, the estimation of the signal and noise subspaces from the STFDs highly improves the angular resolution performance
- STFDs allow DOA estimation of coherent source signals with different time frequency signatures
- STFDs allow to estimate more "DOAs than sensors" (an ongoing research work)

Time frequency array processing



STFD structure in narrow-band propagation environment

- Linear data model in narrow-band propagation environment

$$\mathbf{z}(n) = \mathbf{A}\mathbf{s}(n) + \mathbf{n}(n),$$

  - $\mathbf{z}(n)$: $K \times 1$ data vector received at the antennae
  - $\mathbf{s}(n)$: $L \times 1$ source data vector
  - $\mathbf{A} = [\mathbf{a}_1 \cdots \mathbf{a}_n]$: Propagation matrix
  - $\mathbf{a}_i$: steering vector of the ith source
  - $\mathbf{n}(n)$: noise vector assumed stationary, temporally and spatially white, zero mean, and independent of the source signals.

STFD structure in narrow-band propagation environment

- Expectation of the STFDs:

$$\tilde{\mathbf{D}}_{\mathbf{zz}}(n,k) = E[\mathbf{D}_{\mathbf{zz}}(n,k)] = \mathbf{A}\tilde{\mathbf{D}}_{\mathbf{ss}}(n,k)\mathbf{A}^H + \sigma^2\mathbf{I},$$

- Data covariance matrix "commonly used in array signal processing":

$$\mathbf{R}_{\mathbf{zz}} = E[\mathbf{z}(n)\mathbf{z}(n)^*] = \mathbf{A}\mathbf{R}_{\mathbf{ss}}\mathbf{A}^H + \sigma^2\mathbf{I},$$

STFDs and Covariance matrix exhibit the same eigen-structure

This structure is exploited to estimate some signal parameters through subspace-based techniques

Advantages of STFD over Covariance Matrix

- STFD-based approaches:
  - can handle signals corrupted by interference occupying same frequency band and/or time slot but with different TF signatures
  - are more robust to additive noise due to spreading of noise power while localizing signal energy in TF plane
  - STFDs based algorithms exploit the time-frequency representation of the signals together with the spatial diversity provided by the multi-antennae

Pioneer works

- The concept of the STFD was introduced first in **1996** by A. Belouchrani and M. Amin
- Successfully used in solving the problem of the blind separation of non stationary signals
- Then applied to solve the problem of direction of arrival (DOA) estimation
- Since then, several work including book chapters and special issues were published in this area using the new concept of the STFD

EURASIP Journal on Advances in Signal Processing. Special Issue on Time frequency and array processing of non-stationary signals, 30 October 2012. Lead guest editor: A. Belouchrani. Guest editors: K. Abed-Meraim and B. Boashash

A. Belouchrani, M. G. Amin, N. Thirion-Moreau and Y. D. Zhang, "Source Separation and Localization using Time-frequency Distributions: An overview", IEEE Signal Processing Magazine, November 2013.

Data model for DOA estimation

- Recall the narrow-band Data model

$$\mathbf{z}(n) = \mathbf{A}(\theta)\mathbf{s}(n) + \mathbf{n}(n)$$

  - $\mathbf{A}(\theta) = [\mathbf{a}(\theta_1), \cdots, \mathbf{a}(\theta_L)]^T$
  - $\mathbf{a}(\theta_k)$ and θ_k defines the steering vector
  - θ_k: Direction of arrival (DOA) of the k-th user

Motivations to Time frequency DOA estimation

- DOA estimation: important issue in next generation wireless communications needed for achieving space division multiple access (SDMA) or just localization
- Most popular techniques in traditional array processing: Subspace based methods (e.g. MUSIC)
- These methods assume stationarity while typical signals in wireless communications and radar application, such as frequency hopping signals or frequency modulated signals, are non stationary with some a priori known information on their time varying frequency content

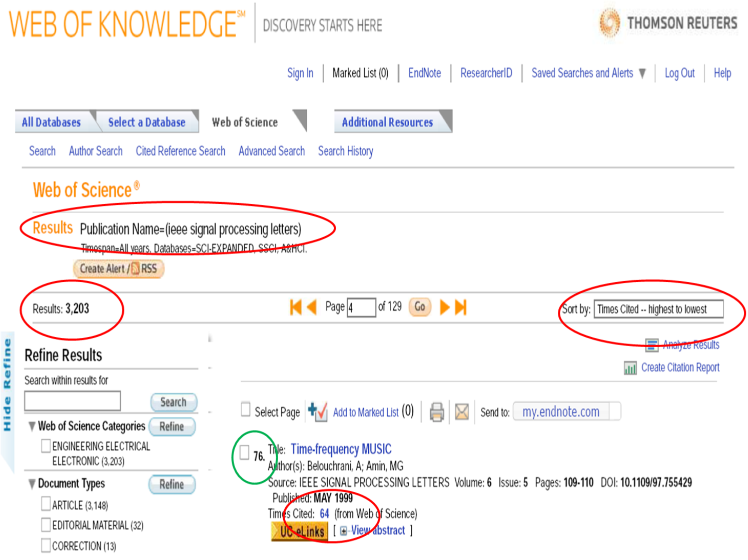
Synergistic relationship between the spatial domain and the time frequency domain

- Several non-spatial features such as time and/or frequency signatures of the signals are ignored in conventional methods
- These defects may result in unaffordable estimation error
- In most wireless communications systems and active radar systems, signals are cooperative and many information contained in these signals are known or can be obtained a priori
- One can exploit these information not only in the spatial domain but also in the time frequency domain in order to improve the performance

Synergistic relationship between the spatial domain and the time frequency domain

- STFDs based DOA estimation methods achieve this synergistic relationship
- Several traditional DOA estimation techniques have been extended to non stationary signals thanks to the use of the STFD instead of the covariance matrix:
  - Time Frequency MUSIC (TF-MUSIC) (Belouchrani et al.)
  - Time Frequency Maximum Likelihood (TF-ML) (Zhang et al.)
  - Time Frequency Signal Subspace Fitting (TF-SSF) (Jin et al.)
  - Time Frequency ESPRIT (TF-ESPRIT) (Hassanien et al.)

Time-Frequency MUSIC

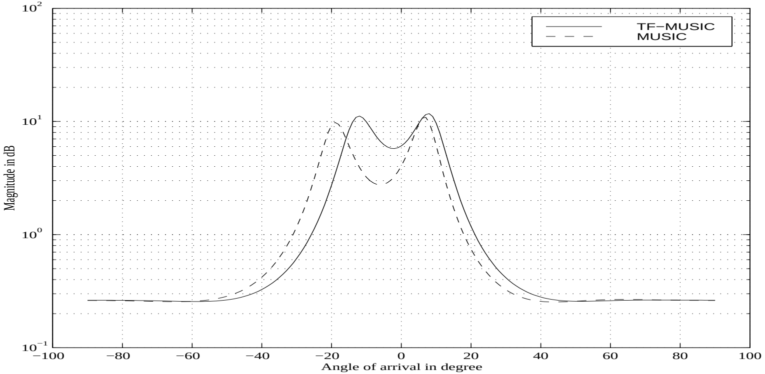


Performance comparison of TF-MUSIC with Conventional MUSIC

- Conventional MUSIC estimates the noise subspace $\mathbf{E}_n$ from the covariance matrix then computes the spatial spectrum: $f(\theta) = |\mathbf{E}_n^H \mathbf{a}(\theta)|^{-2}$
- Time Frequency MUSIC estimates the noise subspace $\mathbf{E}_n$ from the STFDs matrices at auto-terms then computes the spatial spectrum $f(\theta)$
- Setting
    - Uniform linear array of 4 sensors receiving signals from 2 sources located at $\theta_1 = 10^o$ and $\theta_2 = -10^o$.
    - 1st source: chirp signal with frequencies $\omega_1 = 0.1\pi$ $\omega_2 = 0.6\pi$.
    - 2nd source: chirp signal with frequencies $\omega_1' = 0.6\pi$ and $\omega_2' = 0.1\pi$.
    - Zero-mean Gaussian white noise.
    - 50 STFD matrices are considered
    - Variance of $\hat{\theta}$ is estimated over 1000 MC-runs

Simulation results

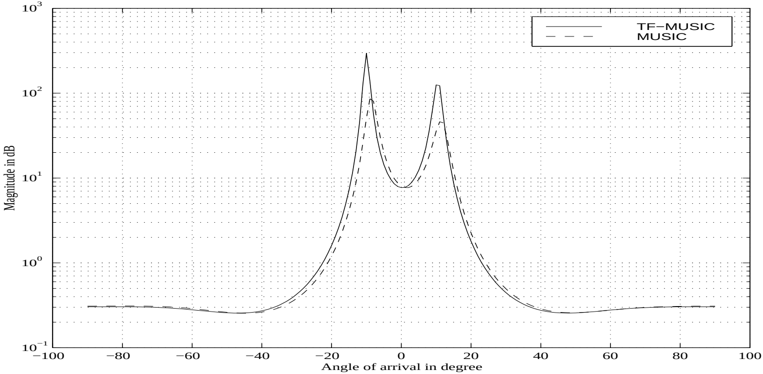
- SNR=0 dB and 100 samples



Source localization.

Simulation results

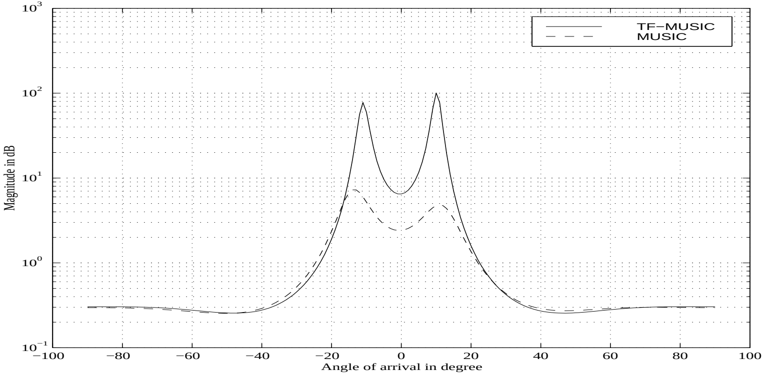
- SNR=10 dB and 100 samples



Source localization.

Simulation results

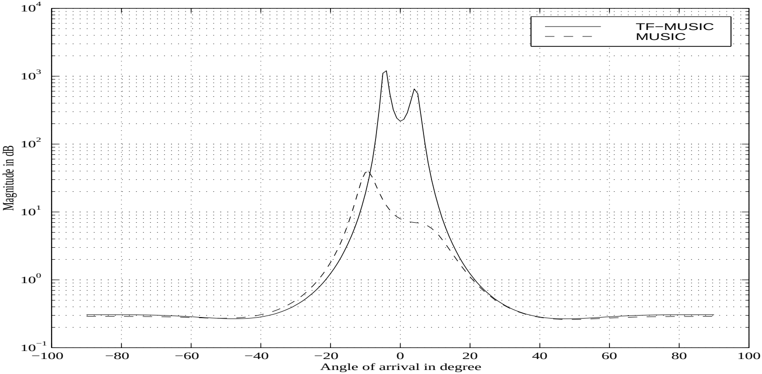
- SNR=5 dB and 100 samples



Source localization.

Simulation results

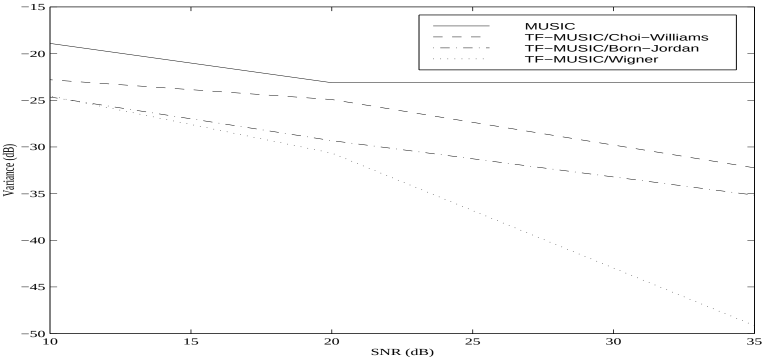
- SNR=5 dB and 50 samples



Source localization.

Performance evaluation

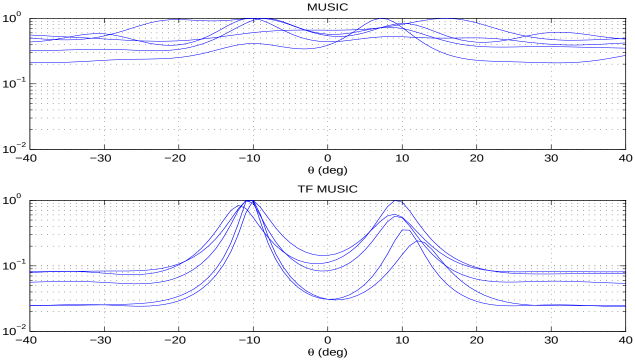
- (1) Choi-Williams (2) Born-Jordan (3) Wigner Kernels



Variance of $\hat{\theta}$ vs SNR

Spatial spectra of MUSIC and TF-MUSIC

- $(\theta_1, \theta_2) = (-10^o, 10^o)$, SNR=-20 dB, 1024 samples and 20 Mc-runs



Time-frequency Maximum Likelihood (ML)

- Time-frequency ML is obtained by solving the minimization problem defined by:

$$\hat{\theta}^{tf} = \arg\min_{\theta}\{Tr\{\Pi_{\mathbf{A}}^{\perp}\hat{\mathbf{D}}_{\mathbf{xx}}\}\}$$

with

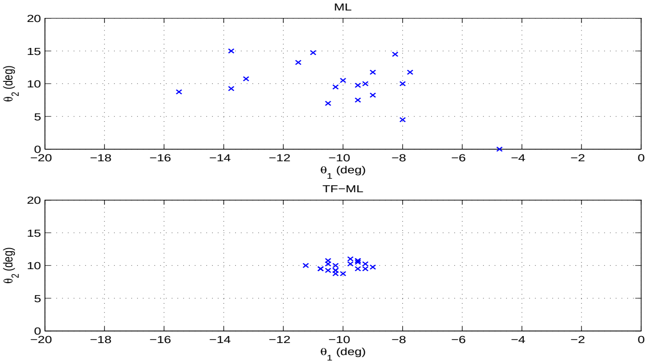
$$\hat{\mathbf{D}}_{\mathbf{xx}} = \frac{1}{K}\sum_{i=1}^{K}\mathbf{D}_{\mathbf{xx}}(t_i, f_i)$$

where $\mathbf{D}_{\mathbf{xx}}(t_i, f_i)$ are STFD matrices selected at (t_i, f_i) auto-term points, $i = 1, 2, ..., K$.

- Conventional ML uses the sampled covariance matrix $\hat{\mathbf{R}}_{\mathbf{zz}}$ instead of the average of the STFDs matrices

Maximization of TF-ML and conventional ML functions

- $(\theta_1, \theta_2) = (-10^o, 10^o)$, SNR=-20 dB, 1024 samples and 20 Mc-runs



Time-frequency subspace fitting: TF-WSF

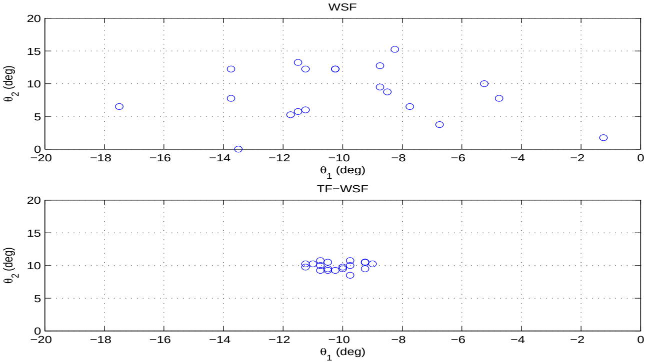
- The Weighted Subspace Fitting (WSF) approach is applied to the signal subspace matrix $\mathbf{E}_s$ estimated from the STFDs matrices $\Longrightarrow$

$$\hat{\theta}_{TF-WSF} = \arg\min_{\theta}\{Tr\{\Pi_{\mathbf{A}}^{\perp}\hat{\mathbf{E}}_s\mathbf{W}\hat{\mathbf{E}}_s^H\}\}$$

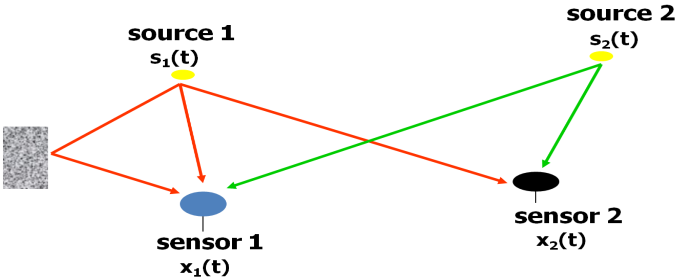
- Conventional WSF estimates the signal subspace matrix $\mathbf{E}_s$ from the sampled covariance matrix $\hat{\mathbf{R}}_{\mathbf{zz}}$ instead of the STFDs matrices.

Minimization of TF-WSF and WSF functions

- $(\theta_1, \theta_2) = (-10^o, 10^o)$, SNR=-20 dB, 1024 samples and 20 Mc-runs.



Blind source separation (BSS) problem



- Different linear combinations of the emitted source signals are received at the sensors
- Blind source separation consists of recovering the source signals from their mixtures with no a priori knowledge on the propagation model

Three Major Approach Classes to BSS

- Independent Component Analysis
- Second Order Statistic Methods
- Time frequency Methods

Independent Component Analysis (ICA)

- Assumes statistical independence of the sources
- Forces the independence of the estimated sources
- Exploits the spatial diversity

Fails if more than one source is Gaussian

Second Order Statistic (SOS) Methods

- Assumes decorrelation of the sources
- Forces the decorrelation across time-delays of the estimated sources
- Exploits the spatial diversity and the spectral diversity

Fails if the sources have identical spectra shapes
In contrast to ICA, SOS can separate Gaussian sources

Time frequency (TF) Methods

- Assumes sources with different time frequency (TF) signatures
- Forces the spatial time frequency distribution structures of the estimated sources
- Exploits the spatial diversity and the time frequency diversity

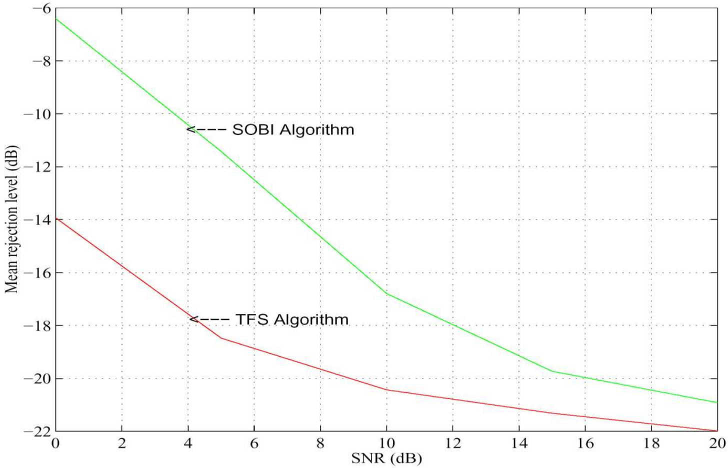
Fails if the sources have identical time frequency signatures
In contrast to ICA and SOS, TF can separate Gaussian sources with identical spectra shape

Some popular approaches

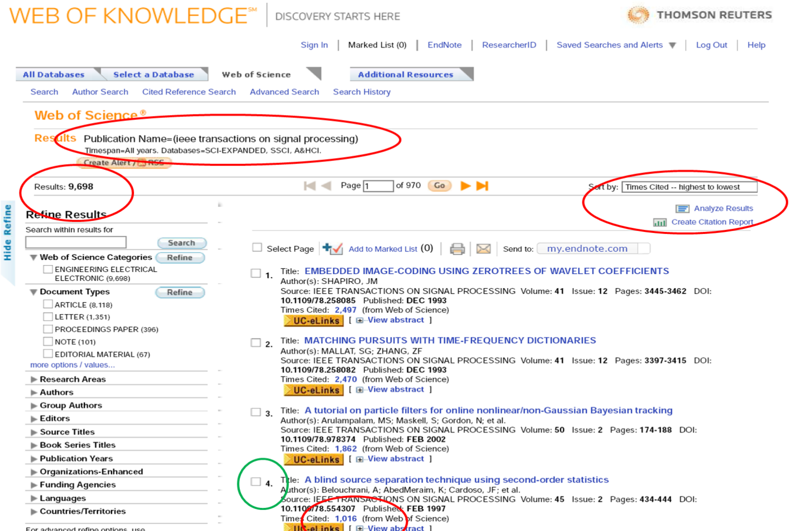
- **JADE**
  J. F. Cardoso and A. Souloumiac, *Blind beamforming for non Gaussian signals*, Proc. IEE, June 1993.
- **SOBI**
  A. Belouchrani, K. Abed-Meraim, J.F. Cardoso and E. Moulines, *A blind source separation technique using second order statistics*, IEEE Trans. on Signal Processing, February 1997.
- **TFBI**
  A. Belouchrani and M. G. Amin, *Blind Source Separation Based on Time-Frequency Signal Representation*, IEEE Trans. on Signal Processing, November 1998.

Simulation results (TFBISOBI)

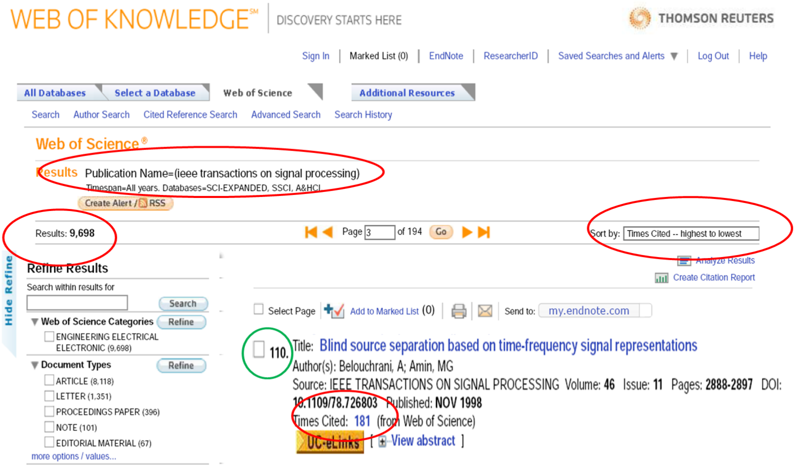
- 2 speech sources, 3 sensors, 6084 samples,100 Mcruns



Second Order Blind Identification (SOBI)



Time-Frequency Blind Identification (TFBI)

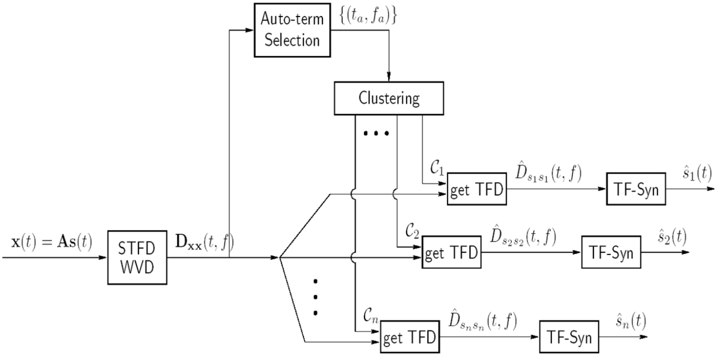


Separating more sources than sensors: A STFDs based approach

The idea

- Select auto-term points where only one source exists, this comes to have the matrix Dzz(n,k) at that point of rank one.
- Cluster together the auto-term points associated to the same principal eigen-vector of Dzz(n, k) representing a particular source signal.
- The estimates of the source signals are obtained from the selected auto terms using a time frequency synthesis algorithm.
- The missing auto-terms in the classification, often due to intersection points, are automatically interpolated in the synthesis process

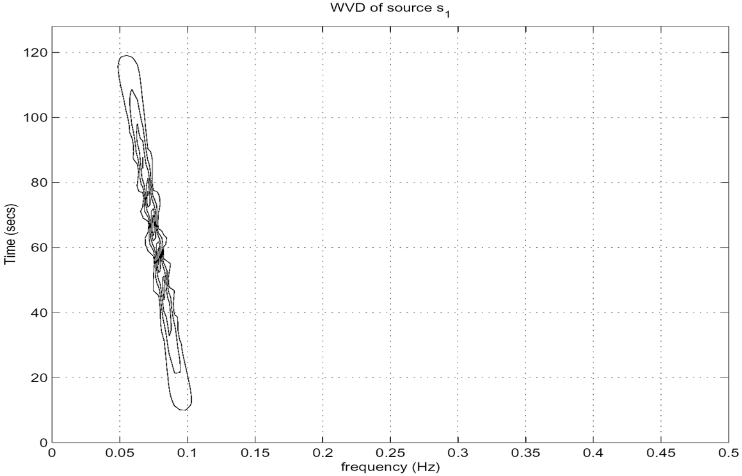
Separating more sources than sensors: A STFDs based approach



N. Linh-Trung, A. Belouchrani, K. Abed-Meraim and B. Boashash, Separating more sources than sensors using time-frequency distributions, EURASIP Journal on Applied Signal Processing, Dec. 2005

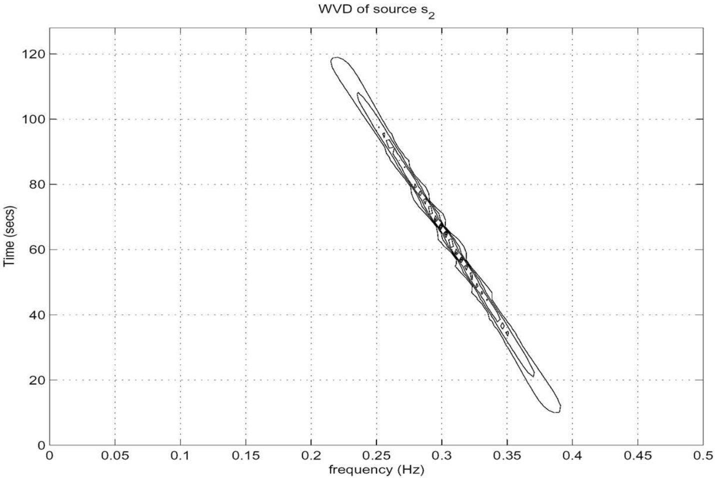
Simulation results

- WVD of the second source



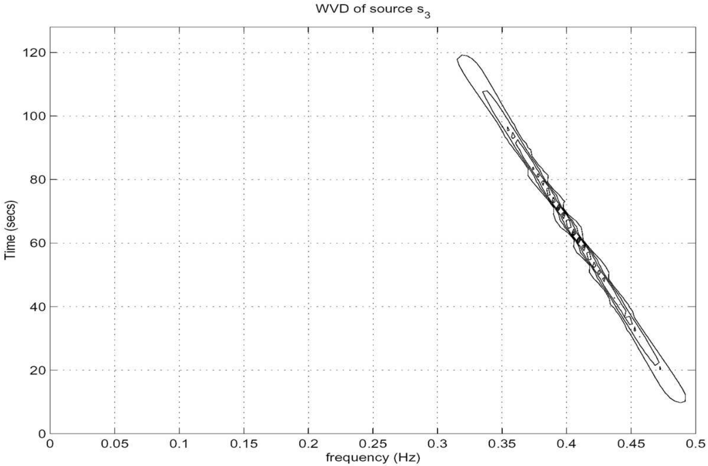
Simulation results

- WVD of the second source



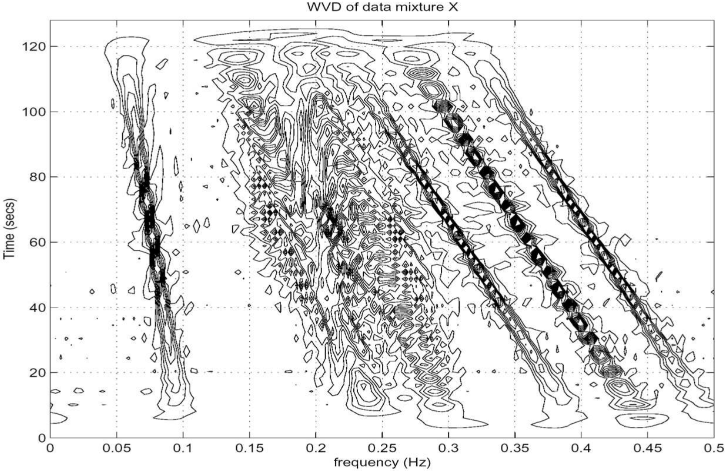
Simulation results

- WVD of the third source



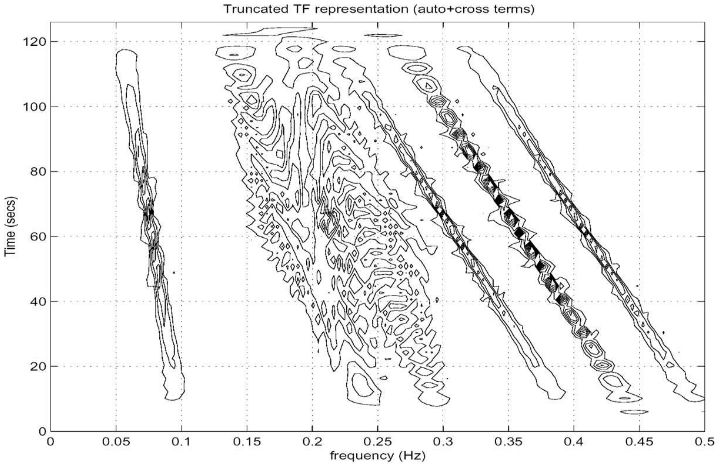
Simulation results

- WVD of the Data mixture



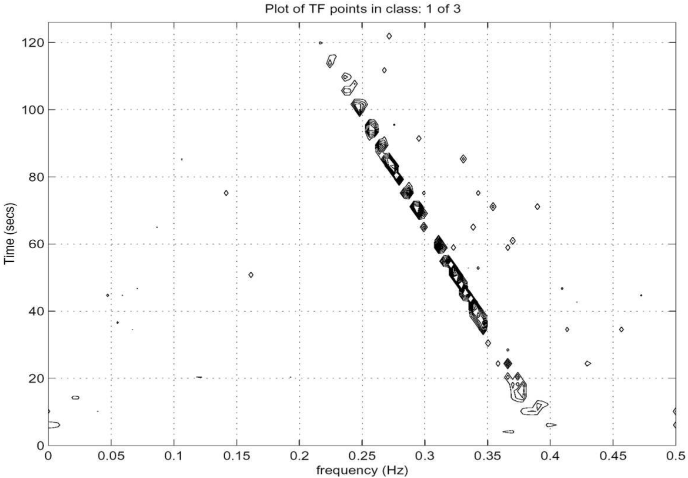
Simulation results

- Truncating the time frequency representation to reduce the number of testing points



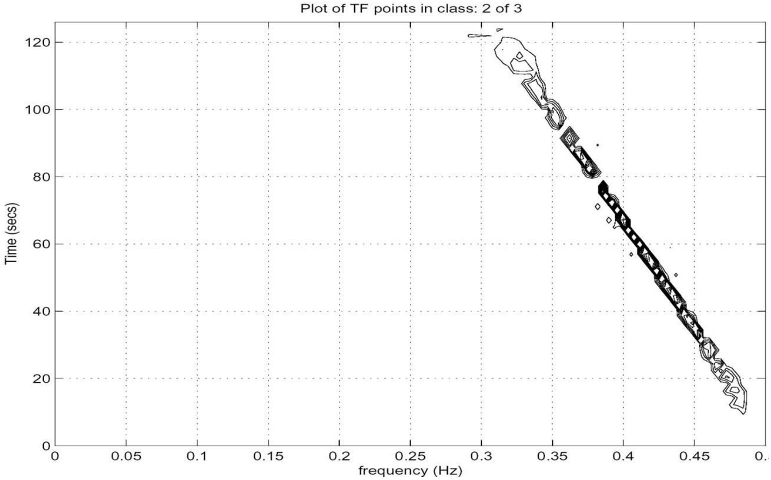
Simulation results

- Classification result: cluster 1



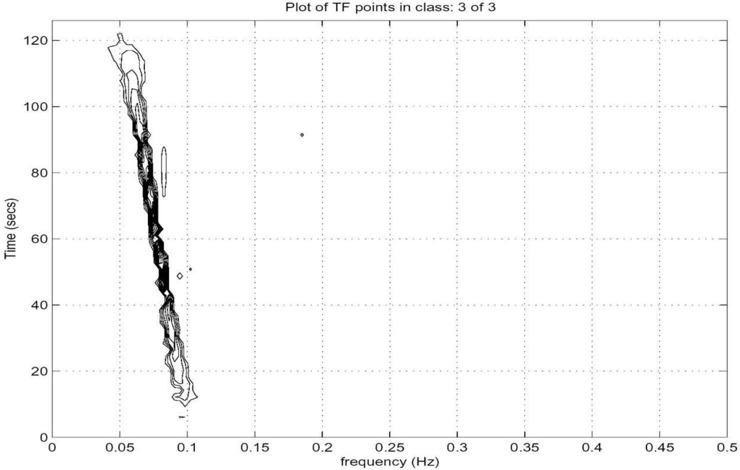
Simulation results

- Classification result: cluster 2



Simulation results

- Classification result: cluster 3



Some Open problems

- Exploitating cross terms in TF array signal processing (Belouchrani et al. 2004-)
- Joint DOD/DOA estimation in MIMO radar exploiting STFDs (Zhang et al. 2012-)
- Application of STFDs for audio source separation (Arberet et al. -2012)
- Performance analysis for time-frequency MUSIC algorithm in presence of both additive noise and array calibration errors (Khodja and Belouchrani 2011-2012)
- DOA estimation of Wideband signal in the time frequency domain (Gershman et al. 2002, Djurovic et al. 2012)
- Astrophysical image separation by blind time-frequency source separation methods (Ozgen et al. 2008-)