Robust Optimization of the Self-scheduling and Market Involvement for an Electricity Producer
Lima, R. M.

2006 - PhD
Department of Chemical Engineering
Faculty of Engineering, University of Porto, Portugal

An integrated strategy for simulation and optimization of chemical processes

Salcedo, R. L. and Barbosa, D.
Lima, R.M.

2006–2008 – Post-doc
Ignacio E. Grossmann
Carnegie Mellon University, PA, USA
Department of Chemical Engineering

Optimal synthesis of p-xylene separation processes based on crystallization

Process synthesis, complex MINLP problems
2008 – 2011 – Researcher
Ignacio E. Grossmann
PPG Industries
Carnegie Mellon University, PA, USA
Department of Chemical Engineering

Planning and long-term scheduling of single stage multi-product continuous lines with a complex recycling structure
Lima, R. M.

2011 – Co-Fund Marie Curie Fellowship

Laboratório Nacional de Energia e Geologia, I.P. (LNEG), Lisbon, Portugal

Project: Planning and Scheduling of Optimal Mix of Renewable Sources in Sustainable Power Systems

- Optimization models and solution methods
- Interdisciplinary work
Lima@KAUST

- Research Scientist, joined SRI-UQ@KAUST on October, 2014
- Working with Omar Knio and Ibrahim Hoteit
- Optimization under uncertainty
- Merge Uncertainty Quantification with Optimization
- Focus on high impact applications

\[
\max_{x,z,y} \quad c^T_x x + c^T_z z + c^T_y y \\
\text{s.t.} \quad Cx \leq b \\
\quad \quad Dz = d \\
\quad \quad Fy + Gx = w \\
\quad \quad Az + By \leq d \\
\quad \quad Hy \leq h \\
\quad \quad x, y \geq 0, z \in \{0,1\},
\]

Application

Uncertainty

Solver

Decomposition

Model
Motivation and objectives

• **Research developments and challenges**
  – Developments in **two stage adaptive Robust Optimization (RO)**
    • Bertsimas and Slim, 2003
    • Bertsimas et al., 2011
    • Bertsimas et al., 2013
    • Thiele et al., 2009

• **Problem features**
  – **Complementarity of energy sources:** hydro and wind
  – **Uncertainty** due to renewable energy sources
  – **Deregulation** of electricity markets
    • Scheduling problems to minimize operational costs
    • Maximize profit by their interaction with the electricity market

Develop an **optimization framework** based on RO to support the decision making of electricity producers in a **market** environment.
RO Overview

• Aims to find a robust solution for a problem under uncertainty
  – Where by robust it is meant that such solution is the optimal for the worst conditions within an uncertainty set describing the uncertainty

• RO advantages
  1. Under specific conditions leads to computational tractable problems
  2. Results can be very reliable, since worst case situations are considered
  3. It does not require a distribution of probabilities

• RO disadvantages
  0.
  1. **Crude** representation of the uncertainty
  2. Solutions can be very conservative

Meaningful uncertainty sets for RO -> Big Data available
Control the conservatism level
Problem definition

- Mixed power generation system operating in an electricity market

- Combinatorial scheduling problem
- Constraints on the technical operation of the units
Decision framework

2\textsuperscript{nd} stage decisions
- \textbf{Thermal plant dispatch}
  - Power output levels subject to commitment
- \textbf{Pump-storage hydro plant dispatch}
  - Power output levels
  - Power consumption to pump
- \textbf{Sell and buy electricity in the pool}
  - 1 week
  - Resolution: 1 hour

1\textsuperscript{st} stage decisions
- \textbf{Self-scheduling}
  - Fix commitment of the thermal unit → Fix 0-1 variables – on/off status of thermal unit
- \textbf{Forward contracting}
  - Sign selling or buying contracts → Decide buy or sell, power and price

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Problem statement

- **Given**
  - Electricity producer with a portfolio of generation units
    - Operating constraints of the units
    - The system can be operated as a virtual power system
  - The producer can interact with the market
    - Buy or sell through forward contracts and the pool
  - The time horizon of 1 week, with the resolution of 1 hour
  - Forward contracts format
  - Electricity price forecasts and error limits
  - Wind power forecast and error limits

- **Determine**
  - Power generation schedule by unit
  - Hourly electricity sold and bought in the pool, and by contracts

- **Maximize**
  - **Operational profit**
2-stage adaptive RO framework

Deterministic model

\[
\begin{align*}
\max_{x, z, y} & \quad c_x^T x + c_z^T z + c_y^T y \\
\text{s.t.} & \quad Cx \leq b \\
& \quad Dz = d \\
& \quad Fy + Gx = w \\
& \quad Az + By \leq d \\
& \quad Hy \leq h \\
& \quad x, y \geq 0, z \in \{0, 1\},
\end{align*}
\]

2-stage adaptive RO

\[
\begin{align*}
\max_{x, z} & \quad c_x^T x + c_z^T z + \mathbf{R}(x, z) \\
\text{s.t.} & \quad Cx \leq b \\
& \quad Dz = d \\
& \quad x \geq 0, z \in \{0, 1\},
\end{align*}
\]

\[
\begin{align*}
\mathbf{R}(x, z) = \min_{w, c_y} \quad & \max_y c_y^T y \\
\text{s.t.} & \quad Fy = w - Gx \\
& \quad By \leq d - Az \\
& \quad Hy \leq h \\
& \quad y \geq 0 \\
\text{s.t.} & \quad w, c_y \in W.
\end{align*}
\]

Multi-period MILP problem, 
\(x, y\) continuous variables 
\(z\) binary variables

Uncertainty on wind power and electricity prices

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Comparison of 2-Stage Formulations

2-stage adaptive RO

\[
\begin{align*}
\max_{x,z} & \quad c_x^T x + c_z^T z + R(x, z) \\
\text{s.t.} & \quad Cx \leq b \\
& \quad Dz = d \\
& \quad x \geq 0, z \in \{0, 1\},
\end{align*}
\]

\[
R(x, z) = \min_{w,c_y} \max_y c_y^T y \\
\text{s.t.} & \quad Fy = w - Gx \\
& \quad By \leq d - Az \\
& \quad Hy \leq h \\
& \quad y \geq 0 \\
\text{s.t.} & \quad w, c_y \in W.
\]

2-stage Stochastic Programming

\[
\begin{align*}
\max_{x,z} & \quad c_x^T x + c_z^T z + R(x, z) \\
\text{s.t.} & \quad Cx \leq b \\
& \quad Dz = d \\
& \quad x \geq 0, z \in \{0, 1\},
\end{align*}
\]

\[
Q(x, \xi(w)) = \max_y c_y(w)^T y \\
\text{s.t.} & \quad Fy = h(w) - G(w)x \\
& \quad By(w) \leq d(w) - A(w)z \\
& \quad Hy(w) \leq g(w) \\
& \quad y(w) \geq 0
\]
2-stage adaptive RO framework (cont.)

Recourse problem

\[ R(x, z) = \min_{w, c_y} \max_y \begin{bmatrix} \mathbf{c}_y^T \mathbf{y} \\ \mathbf{F} \mathbf{y} = w - G \mathbf{x} \\ By \leq d - Az \\ Hy \leq h \\ y \geq 0 \end{bmatrix} \]

s.t. \( w, c_y \in W \).

Inner of the recourse problem

\[ IR(x, z, w, c_y) = \max_y \begin{bmatrix} \mathbf{c}_y^T \mathbf{y} \\ \mathbf{F} \mathbf{y} = w - G \mathbf{x} \\ By \leq d - Az \\ Hy \leq h \\ y \geq 0 \end{bmatrix} \]

Convex LP problem

Assuming strong duality, the dual of IR is given by

\[ DIR(x, z, w, c_y) = \min_{\alpha, \beta, \mu} \begin{bmatrix} (w - G \mathbf{x})^T \alpha + (d - A \mathbf{z})^T \beta + h^T \mu \\ \mathbf{F}^T \alpha + \mathbf{B}^T \beta + \mathbf{H}^T \mu \geq c_y \\ \alpha \in \mathbb{R}, \ \beta \geq 0, \ \mu \geq 0 \end{bmatrix} \]

Next step: Merge the outer problem of the Recourse with the Dual DIR
2-stage adaptive RO framework (cont.)

Reformulated recourse problem

\[ LDR(x, z) = \min_{w,c_y,\alpha,\beta,\mu} (w - Gx)^T \alpha + (d - Az)^T \beta + h^T \mu \]

s.t. \( F^T \alpha + B^T \beta + H^T \mu \geq c_y \)

\( \alpha \in \mathbb{R}, \ \beta \geq 0, \ \mu \geq 0 \)

\( w, c_y \in W. \)

2-stage adaptive RO

\[
\begin{align*}
\max_{x,z} \quad & c_x^T x + c_z^T z \\
+ \min_{w,c_y,\alpha,\beta,\mu} \quad & (w - Gx)^T \alpha + (d - Az)^T \beta + h^T \mu \\
\text{s.t.} \quad & F^T \alpha + B^T \beta + H^T \mu \geq c_y \\
& \alpha \in \mathbb{R}, \ \beta \geq 0, \ \mu \geq 0 \\
& w, c_y \in W
\end{align*}
\]

s.t. \( Cx \leq b \)

\( Dz = d \)

\( x \geq 0, \ z \in \{0, 1\}, \)

This is a **nontrivial optimization problem** because of the **bi-level** structure.

Difficult to solve with a standard solver.
(Dual) Constraint Generation Algorithm

(Thiele et al., 2009; Zhang and Guan, 2009; Jiang et al. 2010; Zugno and Conejo, 2013)

{Initialization}
\[
LB := -\infty, \quad UB := +\infty, \quad k := 1
\]
\[
x^k := x^0, \quad z^k := z^0
\]
\[
O := \emptyset
\]

while \((UB - LB)/LB \leq \varepsilon\) do

{Solve subproblem}
\[
LDR(x^k, z^k) = \min_{w, c_y, \alpha, \beta, \mu} \left( w - Gx^k \right)^T \alpha + \left( d - Az^k \right)^T \beta + h^T \mu
\]
\[
s.t. \quad F^T \alpha + B^T \beta + H^T \mu \geq c_y
\]
\[
\alpha \in \mathbb{R}, \quad \beta \geq 0, \quad \mu \geq 0
\]
\[
w, c_y \in W.
\]

\[
LB := \max\{LB, c_x^T x^k + c_z^T z^k + LDR(x^k, z^k)\}
\]
\[
O := O \cup \{k\}
\]

{Solve Master problem}
\[
PF(x, z) = \max_{x, z, \Theta} \left( c_x^T x + c_z^T z + \Theta \right)
\]
\[
s.t. \quad \Theta \leq \left( w^k - Gx \right)^T \alpha^k + \left( d - Az \right)^T \beta^k + h^T \mu^k, \quad k \in O
\]
\[
Cx \leq b
\]
\[
Dz = d
\]
\[
x \geq 0, \quad z \in \{0, 1\}, \quad \Theta \in \mathbb{R}
\]

\[
UB := \min\{UB, PF(x, z)\}
\]
\[
k := k + 1
\]

end while
Primal Constraint Generation Algorithm

Master Problem

\[
PF(x, z) = \max_{x,z,\Theta} c^T_x x + c^T_z z + \Theta \\
\text{s.t.} \quad \Theta \leq \left( w^k - Gx \right)^T \alpha^k + (d - Az)^T \beta^k + h^T \mu^k, \quad k
\]
\[
C x \leq b \\
D z = d \\
x \geq 0, \ z \in \{0, 1\}, \ \Theta \in \mathbb{R}
\]

Recourse Problem

\[
R(x, z) = \min_{w,c_y} \max_y c^T_y y \\
\text{s.t.} \\
F y = w - Gx \\
B y \leq d - Az \\
H y \leq h \\
y \geq 0
\]

Introduce a copy of the primal variables \(y\)

\[
PF(x, z) = \max_{x,z,\Theta} c^T_x x + c^T_z z + \Theta \\
\text{s.t.} \quad \Theta \leq \left( w^k - Gx \right)^T \alpha^k + (d - Az)^T \beta^k + h^T \mu^k, \quad k \in O \\
\Theta \leq c^T_{y^k} y^k, \quad k \in O \\
F y^k = w^k - Gx, \quad k \in O \\
B y^k \leq d - Az, \quad k \in O \\
H y^k \leq h, \quad k \in O \\
C x \leq b \\
D z = d \\
x \geq 0, \ z \in \{0, 1\}, \ y^k \geq 0, \ \Theta \in \mathbb{R},
\]
Uncertain Polyhedral Sets

- Uncertainty is described by polyhedral sets: built around a nominal value
  - Forecast value
  - Forecast error $\rightarrow$ lower and an upper bound
- This is an alternative approach to a scenario framework built from a probability distribution

$$w_t = \overline{w}_t + z_t^+ w^u_t - z_t^- w^l_t$$

Wind power

- Forecast
- Lower and upper bounds

Electricity pool prices

- Forecast
- Lower and upper bounds
Risk management

• The solution is at one of the extreme points of the convex set
• May lead to over conservative solutions
• In RO risk management is implemented by budget constraints

\[ \sum_t z^+_t + z^-_t \leq \Gamma \]

\( \Gamma \) – Budget parameter
\( z^+_t, z^-_t \) - 0-1 variables

High \( \Gamma \) – high number of periods exhibit deviations from \( \bar{w}_t \) → Conservative approach

Low \( \Gamma \) – low number of periods exhibit deviations from \( \bar{w}_t \) → Risk prone approach

Wind power uncertainty set

\[ W^w = \left\{ w_t \geq 0, z^+_t, z^-_t \in \{0, 1\}, \forall t, : w_t = \bar{w}_t + z^+_t w^u_t - z^-_t w^l_t, \sum_t z^+_t + z^-_t \leq \Gamma \right\} \]

Electricity pool prices uncertainty set

\[ W^\lambda = \left\{ \lambda_t \geq 0, y^+_t, y^-_t \in \{0, 1\}, \forall t, \lambda_t = \bar{\lambda}_t + y^+_t \lambda^u_t - y^-_t \lambda^l_t, \sum_t y^+_t + y^-_t \leq \Gamma \right\} \]
Characterization of the subproblem

\[ DIR(x, z, w, c_y) = \min_{\alpha, \beta, \mu} \ (w - Gx)^T \alpha + (d - Az)^T \beta + h^T \mu \]

\[ s.t. \quad F^T \alpha + B^T \beta + H^T \mu \geq c_y \]
\[ \alpha \in \mathbb{R}, \quad \beta \geq 0, \quad \mu \geq 0 \]

\[ LDR(x, z) = \min_{w, c_y, \alpha, \beta, \mu} \ (w - Gx)^T \alpha + (d - Az)^T \beta + h^T \mu \]

\[ s.t. \quad F^T \alpha + B^T \beta + H^T \mu \geq c_y \]
\[ \alpha \in \mathbb{R}, \quad \beta \geq 0, \quad \mu \geq 0 \]
\[ w, c_y \in W. \]

\[ \text{Profit}_{\text{pool}} = \min \left\{ \sum_t \left\{ \left[ \sum_f \sum_j \left( f_{f,j}^{\text{sell}} - f_{f,j}^{\text{buy}} \right) \right] - w_t \right\} \alpha_t \right\} \]
\[ + \sum_{i \in \mathcal{T}_H} \sum_t \left[ (P_i^u u_{i,t}) \beta_{i,t} + (-P_i^l u_{i,t}) \gamma_{i,t} \right] + \sum_{i \in \mathcal{T}_H} \sum_{t=1} \left[ (P_0 + RU_i U_{0_i} + SU_i u_{i,t}^{up}) \zeta_{i,t} \right] \]
\[ + \sum_{i \in \mathcal{T}_H} \sum_{t>1} \left[ (RU_i u_{i,t-1} + SU_i u_{i,t}^{up}) \eta_{i,t} + (RD_i u_{i,t} + SD_i u_{i,t}^{dn}) \vartheta_{i,t} \right] \]
\[ + \sum_{i \in \mathcal{H}_Y} \sum_{t=1} \left[ (V_{0_i} + GQ_i^{in}) \mu_{i,t} \right] + \sum_{i \in \mathcal{H}_Y} \sum_{t>1} (GQ_i^{in} \nu_{i,t}) \]
\[ + \sum_{i \in \mathcal{H}_Y} \sum_t \left( Q_i^u \omega_{i,t} + Q_i^u \rho_{i,t} - V_i^l \tau_{i,t} + V_i^u \varphi_{i,t} \right) - \sum_{i \in \mathcal{H}_Y} \sum_{t=t_f} (V_i^E \varphi_{i,t}) \]
Characterization of the subproblem (cont.)

\[ LDR(x, z) = \min_{w, c_y, \alpha, \beta, \mu} (w - Gx)^T \alpha + (d - Az)^T \beta + h^T \mu \]
\[ s.t. \quad F^T \alpha + B^T \beta + H^T \mu \geq c_y \]
\[ \alpha \in \mathbb{R}, \quad \beta \geq 0, \quad \mu \geq 0 \]
\[ w, c_y \in W. \]
\[ -\alpha_t \geq \lambda_t \quad \forall t \]
\[ \alpha_t \geq -\lambda_t \quad \forall t \]

\[ \sum_{i \in TH} p_{i,t} + \sum_{i \in HY} ptb_{i,t} + p_{t}^{buy} + \sum_{f} \sum_{j} f_{f,j}^{buy} + w_t = \sum_{i \in HY} pp_{i,t} + p_{t}^{sell} + \sum_{f} \sum_{j} f_{f,j}^{sell}, \quad \forall t, \]

\[ \nu_{i,t} - \nu_{i,t+1} - \tau_{i,t} + \varsigma_{i,t} \geq 0 \quad \forall i \in HY, t > 1, t < tf \]
\[ \nu_{i,t} - \varphi_{i,t} - \tau_{i,t} + \varsigma_{i,t} \geq 0 \quad \forall i \in HY, t = tf \]
\[ G\mu_{i,t} - K_i^p H_i \xi_{i,t} + \omega_{i,t} \geq 0 \quad \forall i \in HY, t = 1 \]
\[ G\nu_{i,t} - K_i^p H_i \xi_{i,t} + \omega_{i,t} \geq 0 \quad \forall i \in HY, t > 1 \]
\[ \alpha_t + \xi_{i,t} \geq 0 \quad \forall i \in HY, t \]
\[ -\alpha_t + \pi_{i,t} \geq 0 \quad \forall i \in HY, t \]
\[ -G\mu_{i,t} - K_i^p H_i \pi_{i,t} + \rho_{i,t} \geq 0 \quad \forall i \in HY, t = 1 \]
\[ -G\nu_{i,t} - K_i^p H_i \pi_{i,t} + \rho_{i,t} \geq 0 \quad \forall i \in HY, t > 1 \]
Linearization of the subproblem

Linearization of $w_t \alpha_t$

Definition of $w_t$

$$m_t = m_0 + z^+ m_u - z^- m_l$$

$$w_t \alpha_t = w_0 \alpha_t + z^+_t w_t \alpha_t - z^-_t w_t \alpha_t, \quad \forall t,$$

$$\alpha_t \geq - (\lambda_t + y^+_t \lambda^u_t - y^-_t \lambda_l_t), \quad \forall t,$$

Substitution

$$v^+_t \leq - (\lambda_t - \lambda'_t) z^+_t, \quad \forall t,$$

$$v^+_t \leq \alpha_t + (\lambda_t + \lambda^u_t) (1 - z^+_t), \quad \forall t,$$

Linearization

$$v^-_t \geq - (\lambda_t + \lambda^u_t) z^-_t, \quad \forall t,$$

$$v^-_t \geq \alpha_t, \quad \forall t,$$

$$v_t \leq \lambda_3 z_t, \quad \forall t,$$

$$v^-_t \geq \alpha_t - M_4 (1 - z^-_t), \quad \forall t,$$

Based on

$$- \alpha_t \geq \lambda_t \quad \forall t$$

$$\alpha_t \geq - \lambda_t \quad \forall t$$

$$\lambda_t = \lambda_t + y^+_t \lambda^u_t - y^-_t \lambda'_t,$$
Remarks

1. Master and Sub-Problem are MILP problems.

2. The Sub-Problem is always bounded for any first stage decisions (complete recourse) if the option to buy energy from the pool is considered.

3. If the MILP Sub-Problem is not solved to optimality then
   
   I. The LB is not computed with the best solution of the Sub-Problem found, but with the best MILP LB, $\overline{LDR}(x^k, z^k)$

   $$LB := \max\{LB, c_x^T x^k + c_z^T z^k + \overline{LDR}(x^k, z^k)\}$$

   II. The integer solution obtained is still a valid bound

   $$\Theta \leq LDR(x^k, z^k) \leq \overline{LDR}(x^k, z^k)$$
Results

- Computational experiments
  - Case 1
    - 1 thermal unit named G1
    - 1 pumped-storage hydro unit
    - 1 wind farm
  - Case 2
    - 1 thermal unit named G2
    - 1 pumped-storage hydro unit
    - 1 wind farm
  - 2 Algorithms: Dual and Primal
  - 3 Instances of electricity prices: EP1, EP2, EP3
  - Risk management: 5 values for the budget parameter

- Models implemented in GAMS, on a computer with an Intel Core i7@3.07GHz CPU, 64 bits, and 8Gb of RAM. The MILP problems are solved with CPLEX 12.5.
Case 1 – Computational results

Maximum CPU time set to 1500s and 0.1% gap.

<table>
<thead>
<tr>
<th>EP</th>
<th>$\Gamma$</th>
<th>$P$ (€)</th>
<th>Gap (%)</th>
<th># Iter</th>
<th>CPU$^1$ (s)</th>
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<tbody>
<tr>
<td>EP1</td>
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Dual constraint generation

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<th>Constraint generation</th>
<th>$\gamma$ (%)</th>
<th># Iter</th>
<th>CPU$^2$ (s)</th>
<th>$\Delta$CPU (%)</th>
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$\Delta$CPU (%) = \frac{(CPU^2 - CPU^1)}{CPU^1}$
### Case 2 – Computational results

### Dual constraint generation

<table>
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<tr>
<th>EP</th>
<th>$\Gamma$</th>
<th>$P$ (€)</th>
<th>Gap (%)</th>
<th># Iter</th>
<th>CPU(^1) (s)</th>
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<th># Iter</th>
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<th>Gap (%)</th>
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\[ CPU = (CPU^2 - CPU^1)/CPU^1. \]
Convergence profiles: Dual vs Primal

Case 2, $\Gamma = 150$, EP3

16.90% gap
Convergence profiles: Dual vs Primal

Case 2, $\Gamma = 150$, EP1
Both algorithms do not converge

The Primal Constraint Generation Algorithm cannot close the gap
MILP Sub-Problem is not solved to optimality
Case 1, EP3 – Scheduling and Market Results

$$\Gamma = 10, \text{ Risk prone approach}$$

$$\Gamma = 100, \text{ Conservative approach}$$

A more conservative approach:
Decreases the power sold in the pool
Increases the power sold by contract
Risk management results: budget parameter

<table>
<thead>
<tr>
<th>Case 1</th>
<th>EP1</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>$FC$ (MW)</td>
<td>$P_{sell}$ (MWh)</td>
<td>$P_{buy}$ (MWh)</td>
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<tr>
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<td>168</td>
<td>300</td>
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</table>

More conservative approaches:
- Decreases the power sold in the pool
- Increases the power sold by contract

<table>
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<th>EP1</th>
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<tbody>
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<td>$FC$ (MW)</td>
<td>$P_{sell}$ (MWh)</td>
<td>$P_{buy}$ (MWh)</td>
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<td>100</td>
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<tr>
<td>168</td>
<td>82</td>
<td>2,434</td>
<td>4,857</td>
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</table>

More conservative approaches:
- Decreases the total energy

SRI UQ Annual Meeting 2015
Case 2, EP1 – Perfect information for Wind

$\Gamma = 10$, Risk prone approach

$\Gamma = 168$, Conservative approach

A more conservative approach:

- Decreases the power sold in the pool
- Increases the power sold by contract

Different profile for hydro generation
## Risk management results: budget parameter

### Case 1

<table>
<thead>
<tr>
<th>$\Gamma$ (MW)</th>
<th>$FC$ (MW)</th>
<th>$P_{sell}$ (MWh)</th>
<th>$P_{buy}$ (MWh)</th>
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</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>103,756</td>
<td>0</td>
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<td>300</td>
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</tr>
<tr>
<td>168</td>
<td>300</td>
<td>53,356</td>
<td>0</td>
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</table>

**More conservative approaches:**
- Decreases the power sold in the pool.
- Increases the power sold by contract.

### Case 2

<table>
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<th>$\Gamma$ (MW)</th>
<th>$FC$ (MW)</th>
<th>$P_{sell}$ (MWh)</th>
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<tr>
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<td>218</td>
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</table>

**More conservative approaches:**
- Decreases the power sold in the pool.
- Increases the power sold by contract.
Case 2 - Budget parameter vs contract selection

\( \Gamma = 10, \text{ Risk prone approach} \)

\( \Gamma = 168, \text{ Conservative approach} \)
Conclusions and final remarks

• Robust optimization framework
  1. The Sub-Problem has **full recourse** as long as the producer has the **option to buy electricity**, this simplifies the algorithm.

  2. The **two variants** of the constraint generation algorithm have a **similar performance** with exception for some cases where the **Primal version is better**.

  3. Some **MILP** Sub-Problesms are **not solved to optimality**
     I. The constraint generation algorithm **does not close the gap**
     II. The convergence profile **seems** to indicate that it has obtained the optimal solution
Conclusions and final remarks (cont.)

- **Risk management**
  1. Uncertainty only in electricity prices
     - More conservative approaches lead to lower profits (as expected)
     - Selection of forward contracts to hedge against the volatility of the pool
  2. Uncertainty on electricity prices and wind power
     - More conservative approaches lead to lower profits (as expected)
     - It is difficult to foresee and isolate the relation between the conservatism level and the contract selection and pool involvement

Acknowledgements

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Robust optimization applied to energy systems