Generalized Area Spectral Efficiency: An Effective Performance Metric for Green Wireless Communications

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2. ASE of Cellular Network
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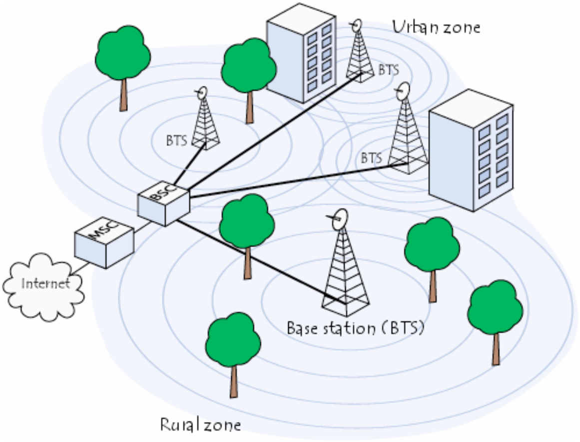
4. GASE of Cooperative Relay Network
5. GASE for Underlay Cognitive Radio Transmission
6. Conclusion

Background

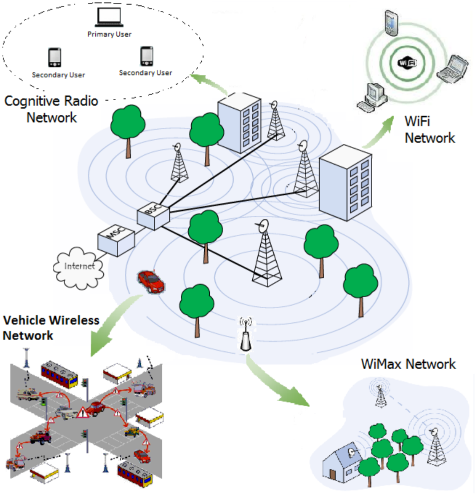
- Wireless communication systems are having increasingly significant ecological impact.
- Future systems need to support the growing data traffics with high spectral efficiency and energy efficiency.
- Various transmission strategies are being implemented, including cooperative relay, femtocell system, and cognitive radio.

Conventional cellular network



- Centralized architecture.
- Large coverage area.
- Sparse and regular frequency reuse.

Emerging cellular networks



- Hybrid network architecture.
- Smaller coverage area.
- Dense and irregular frequency reuse.

Motivation

How to effectively quantify the *spatial* spectrum utilization efficiency?

- Most conventional performance metrics focus on point-to-point link, e.g.
    - Ergodic capacity quantifies bandwidth utilization efficiency.
    - Average error rate evaluates transmission reliability.
- The spatial ‘footprint’ of radio transmission was seldom taken into consideration.
    - Pollute a certain area over its operating spectrum.
    - Simultaneous transmission over this spectrum not possible due to heavy mutual interference.

Area Spectral Efficiency for Cellular Networks

- First introduced by [Alouini/Goldsmith'TVT99] for cellular network.
- Ratio of maximum data rate per unit bandwidth of arbitrary user in BS's coverage area over the size of reuse partition, i.e.

$$ASE = \frac{\overline{C}}{\pi D^2/4},$$

  where D is the reuse distance.
- Recently applied to performance characterization of two-tier cellular network in [Kim et. al.'TVT10].
- Typical hexagon cell structure greatly facilitates ASE analysis.

We generalize the ASE concept to analyze arbitrary wireless systems!

Generalized Area Spectral Efficiency

Ratio of ergodic capacity of the link over the size of the affected area of the radio transmission, i.e. $\eta = \overline{\mathrm{C}}/\mathrm{A}_{\mathrm{aff}}$.

- Affected area $\mathbf{A}_{\mathrm{aff}}$: area where significant amount of transmission power is observed.
- Given a predetermined minimum received signal power $\mathcal{P}_{\mathrm{min}}$, the affected area can be estimated as

$$\mathbf{A}_{\mathrm{aff}} = \int_0^\infty \Pr[\mathcal{P}_{\mathrm{rec}} \geq \mathcal{P}_{\mathrm{min}}] r \, \mathrm{d}r.$$

- Ergodic capacity $\overline{\mathrm{C}}$: averaging the instantaneous link capacity over the distribution of received SNR/SINR Γ

$$\overline{\mathrm{C}} = \int_0^\infty \log_2 \left(1 + \Gamma\right) \mathrm{d}F_\Gamma(\gamma).$$

GASE for Point-to-Point Transmission

- Assume log-distance path loss plus Rayleigh fading environment.
- Incremental area of distance d from the transmitter is affected if and only if

$$P_t \cdot Z/d^a \geq P_{\min},$$

  where P_t is transmission power, a is path loss exponent, and Z is Exponentially distributed random fading power gain.

- The affected area can be determined as

$$\mathrm{A_{aff}} = \frac{1}{a}\Gamma\left(\frac{2}{a}\right)\left(\frac{P_t}{P_{\min}}\right)^{2/a}.$$

- The ergodic capacity of the point-to-point link is

$$\overline{\mathrm{C}} = \frac{1}{\ln 2}\,\mathrm{E}_1\left(\frac{d^a N}{P_t}\right)\exp\left(\frac{d^a N}{P_t}\right).$$

  where N is the noise power.

Numerical Example

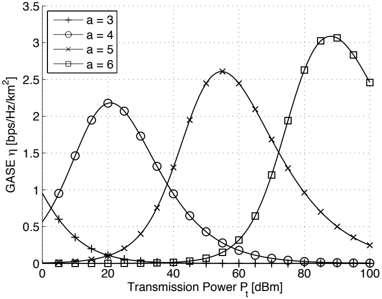


Figure: The effect of transmission power P_t on η.

- Non-monotonic function of P_t.
- Limiting behavior of η.

$$\lim_{P_t \to 0+} \eta = \begin{cases} \infty, & a < 2; \\ 2\log_2 e \cdot \frac{P_{\min}}{Nd^2}, & a = 2; \\ 0, & a > 2, \end{cases}$$

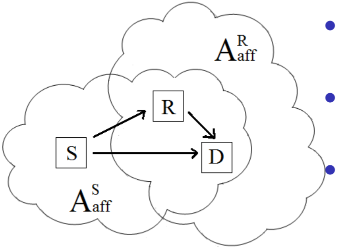
and

$$\lim_{P_t \to \infty} \eta = 0.$$

- Optimal P_t value exists by solving

$$\left(\frac{d^a N}{P_t^*} + \frac{2}{a}\right) \mathrm{E}_1\left(\frac{d^a N}{P_t^*}\right) \exp\left(\frac{d^a N}{P_t^*}\right) = 1.$$

ASE of Cooperative Relay Network



- Half-duplex decode-and-forward (DF) & amplify-and-forward (AF) relaying.
- Instantaneous channel capacity based transmission mode selection.
- Different affected areas for source and relay transmission steps in general

$$\mathbf{A}_{\text{aff}}^{\text{S}} = \frac{1}{a}\Gamma\left(\frac{2}{a}\right)\left(\frac{P_S}{P_{\min}}\right)^{2/a}, \quad \mathbf{A}_{\text{aff}}^{\text{R}} = \frac{1}{a}\Gamma\left(\frac{2}{a}\right)\left(\frac{P_R}{P_{\min}}\right)^{2/a}$$

where P_S and P_R are the transmission power of source and relay nodes.

Transmission Mode Selection

- Instantaneous capacity of direct transmission

$$C_d = \log_2(1 + \Gamma_{SD}).$$

- Instantaneous capacity of relay transmission

$$C_r = \frac{1}{2}\log_2(1 + \Gamma_r).$$

- Probability that system performs direct transmission

$$\mathbf{P}_{\text{direct}} = \Pr\left\{C_d > C_r\right\} = \Pr\left\{\Gamma_{\text{SD}}^2 + 2\Gamma_{\text{SD}} > \Gamma_{\text{r}}\right\}$$

- Probability that system performs relay transmission

$$\mathbf{P}_{\text{relay}} = 1 - \mathbf{P}_{\text{direct}}.$$

Ergodic Capacity Analysis

- Instantaneous capacity of cooperative relay system

$$\begin{aligned}\mathbf{C}_{\text{inst}} &= \max\left\{C_d, C_r\right\}\\ &= \frac{1}{2}\log_2\left\{1+\max\left\{\Gamma_{\text{SD}}^2+2\Gamma_{\text{SD}}, \Gamma_{\text{r}}\right\}\right\},\end{aligned}$$

$\Gamma \triangleq \max\left\{\Gamma_{\text{SD}}^2+2\Gamma_{\text{SD}}, \Gamma_{\text{r}}\right\}$ is the equivalent received SNR.

- Ergodic capacity under direct transmission mode

$$\overline{\mathbf{C}}_{\text{d}} = \int_0^\infty \frac{1}{2}\log_2\left(1+\gamma\right) \mathrm{d}F_\Gamma(\gamma \mid \Gamma_{\text{SD}}^2+2\Gamma_{\text{SD}} > \Gamma_{\text{r}}).$$

Need the distribution of Γ conditioning on $\Gamma_{\text{SD}}^2+2\Gamma_{\text{SD}} > \Gamma_{\text{r}}$.

- Ergodic capacity under relay transmission mode, $\overline{\mathbf{C}}_{\text{r}}$, can be similarly obtained.

Conditional pdf of Γ under direct transmission mode

- Conditional pdf of Γ with DF relay protocol

$$f_{\Gamma^{\mathrm{DF}}}(\gamma \mid \Gamma_{\mathrm{SD}}^2 + 2\Gamma_{\mathrm{SD}} > \Gamma_{\mathrm{r}}^{\mathrm{DF}}) = \frac{\overline{\gamma}_{\mathrm{SD}} \cdot f_{\Gamma_{\mathrm{SD}}}(\xi) \cdot F_{\Gamma_{\mathrm{r}}^{\mathrm{DF}}}(\gamma)}{2(\xi+1)\cdot(\overline{\gamma}_{\mathrm{SD}} - \mathfrak{D}(\infty;\alpha_1,\alpha_2))},$$

where $\Gamma_{\mathrm{r}}^{\mathrm{DF}} = \min\{\Gamma_{\mathrm{SR}}, \Gamma_{\mathrm{RD}}\}$, $\xi = \sqrt{\gamma+1} - 1$,
$\alpha_1 = \frac{1}{\overline{\gamma}_{SR}} + \frac{1}{\overline{\gamma}_{RD}}$, $\alpha_2 = \frac{2}{\overline{\gamma}_{SR}} + \frac{2}{\overline{\gamma}_{RD}} + \frac{1}{\overline{\gamma}_{SD}}$,
$\mathfrak{D}(x;\alpha_1,\alpha_2) = \frac{1}{2}\sqrt{\frac{\pi}{\alpha_1}} e^{\frac{\alpha_2^2}{4\alpha_1}} \left[\mathrm{erf}(\sqrt{\alpha_1}\cdot x + \frac{\alpha_2}{2\sqrt{\alpha_1}}) - \mathrm{erf}(\frac{\alpha_2}{2\sqrt{\alpha_1}})\right]$.

- Conditional pdf of Γ with AF relay protocol

$$f_{\Gamma_{\mathrm{r}}^{\mathrm{AF}}}(\gamma) = 2\beta_1 \gamma e^{-\beta_2\gamma}\left\{\beta_2\, \mathrm{K}_1(2\beta_1\gamma) + 2\beta_1\, \mathrm{K}_0(2\beta_1\gamma)\right\},$$

where $\Gamma_{\mathrm{r}}^{\mathrm{AF}} = \frac{\Gamma_{\mathrm{SR}}\cdot\Gamma_{\mathrm{RD}}}{\Gamma_{\mathrm{SR}}+\Gamma_{\mathrm{RD}}}$, $\beta_1 = \frac{1}{\sqrt{\overline{\gamma}_{\mathrm{SR}}\cdot\overline{\gamma}_{\mathrm{RD}}}}$, $\beta_2 = \frac{1}{\overline{\gamma}_{\mathrm{SR}}} + \frac{1}{\overline{\gamma}_{\mathrm{RD}}}$,
$\beta_3 = \frac{1}{\overline{\gamma}_{\mathrm{SD}}} + \frac{1}{\overline{\gamma}_{\mathrm{SR}}} + \frac{1}{\overline{\gamma}_{\mathrm{RD}}}$,
$\mathfrak{A}(x;\beta_1,\beta_3) = \int_0^x 2\beta_1(t^2+2t)e^{-\beta_3(t^2+2t)}\, \mathrm{K}_1(2\beta_1(t^2+2t))\, \mathrm{d}t$.

Overall ASE

$$\mathrm{ASE} = \mathbf{P}_{\mathrm{direct}} \cdot \frac{\overline{\mathbf{C}}_{\mathrm{d}}}{\mathbf{A}_{\mathrm{aff}}^{\mathrm{S}}} + \mathbf{P}_{\mathrm{relay}} \cdot \frac{1}{2} \left(\frac{\overline{\mathbf{C}}_{\mathrm{r}}}{\mathbf{A}_{\mathrm{aff}}^{\mathrm{S}}} + \frac{\overline{\mathbf{C}}_{\mathrm{r}}}{\mathbf{A}_{\mathrm{aff}}^{\mathrm{R}}} \right),$$

- $\mathbf{C}_{\mathrm{d}}$, $\mathbf{C}_{\mathrm{r}}$: ergodic capacity under direct/relay transmission mode.
- $\mathbf{P}_{\mathrm{direct}}, \mathbf{P}_{\mathrm{relay}}$: probability of direct/relay transmission.
- $\mathbf{A}_{\mathrm{aff}}^{\mathrm{S}}$: affected area of source-destination transmission.
- $\mathbf{A}_{\mathrm{aff}}^{\mathrm{R}}$: affected area of relay-destination transmission.

Optimal Relay Locations

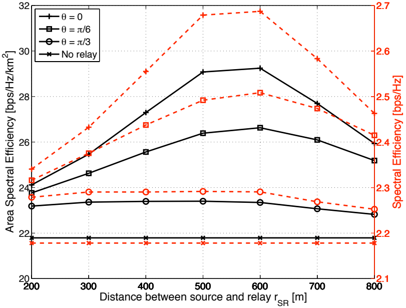


Figure: The effect of the distance between source and relay node r_{SR} on ASE and spectral efficiency with DF relay protocol for different angle θ.

- Optimal relay location is the midpoint between source and destination when $\theta = 0$.
- When θ is small, ASE varies dramatically as the position of relay changes.
- When θ is large, the distance r_{SR} has little effect on ASE.
- Similar observation can be observed for AF-based relay networks.

Effect of Source Transmission Power

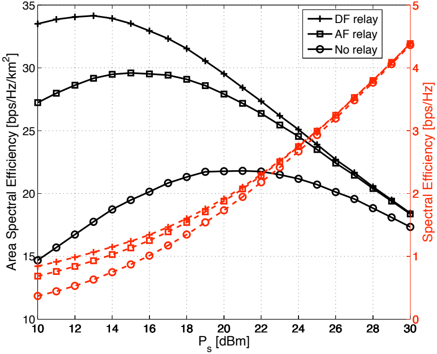


Figure: The effect of the source node transmission power $\mathcal{P}_s$ on ASE with DF and AF relay protocol.

- Cooperative relaying achieves better ASE than conventional network.
  - Opportunistic transmission mode selection.
  - Smaller affected area.
- Optimal transmit power $\mathcal{P}_s$ to maximize ASE exists.
- Increasing the transmission power can lead to a higher spectral efficiency but NOT necessarily increase ASE.

GASE for Underlay Cognitive Radio Transmission

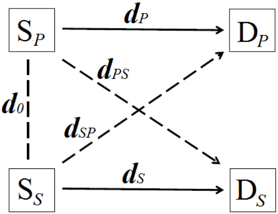


Figure: System model of underlay cognitive radio transmission.

Secondary transmitter S_S opportunistically transmits to secondary receiver D_S using the same frequency bandwidth of primary transmission S_P-D_P as long as the interference constraint on D_P is satisfied.

Parallel Transmission Scenario

- Parallel secondary transmission occurs if the experienced interference power at D_P is less than the threshold I_{th}, i.e. $P_2 \cdot Z / d_{SP}^a < I_{th}$, where P_2 is the transmission power of S_S.
- Affected area with parallel secondary transmission

$$\mathrm{A}_{CR}^{pt} = \int_{\Omega} \mathbb{P}\left\{ P_r(r_p) + P_r(r_s) \geq P_{\min} \right\} \mathrm{d}\Omega,$$

  where r_p and r_s are the distances of the incremental area to the primary transmitter and secondary transmitter, respectively.

- Ergodic capacity

$$\overline{\mathrm{C}}_{CR} = \underbrace{\int_0^{\infty} \log_2(1+\gamma) \cdot \mathrm{d}F_{\Gamma_p}(\gamma)}_{\overline{\mathrm{C}}_{CR}^{p}} + \underbrace{\int_0^{\infty} \log_2(1+\gamma) \cdot \mathrm{d}F_{\Gamma_s}(\gamma)}_{\overline{\mathrm{C}}_{CR}^{s}},$$

  - $\overline{\mathrm{C}}_{CR}^{p}$, $\overline{\mathrm{C}}_{CR}^{s}$: ergodic capacity of primary and secondary transmission.
  - Γ_p, Γ_s: received SINR at D_P and D_S.

Overall GASE

- GASE when parallel transmission occurs

$$\eta_{CR}^{pt} = \frac{\overline{\mathrm{C}}_{CR}^{p} + \overline{\mathrm{C}}_{CR}^{s}}{\mathrm{A}_{CR}^{pt}}.$$

- When $P_2 \cdot Z/d_{SP}^a \geq I_{th}$, the transmission scenario reduces to point-to-point primary transmission only case, with GASE given by

$$\eta_{CR}^{st} = \frac{\frac{1}{\ln 2}\,\mathrm{E}_1\left(\frac{d^a N}{P_1}\right)\exp\left(\frac{d^a N}{P_1}\right)}{\frac{1}{a}\Gamma\left(\frac{2}{a}\right)\left(\frac{P_1}{P_{\min}}\right)^{2/a}},$$

  where P_1 is the primary source transmission power.

- Overall GASE of underlay cognitive transmission

$$\eta_{CR} = \mathcal{P} \cdot \eta_{CR}^{pt} + (1 - \mathcal{P}) \cdot \eta_{CR}^{st},$$

  where $\mathcal{P} = \mathbb{P}\left\{P_2 \cdot Z/d_{SP}^a < I_{th}\right\}$.

Numerical Examples

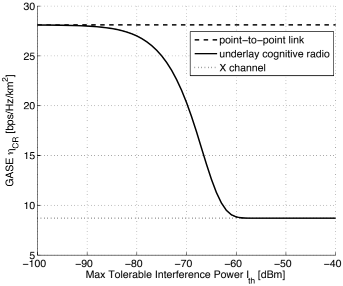


Figure: The effect of the max tolerable interference power I_{th} on GASE.

GASE of underlay cognitive radio transmission include those of the point-to-point transmission and X channel transmission as special case.

- When $I_{th} \rightarrow 0$, converge to the point-to-point transmission case.
- When $I_{th} \rightarrow \infty$, converge to the X channel case.

Numerical Examples

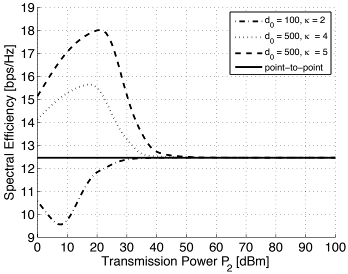


Figure: Spectral Efficiency

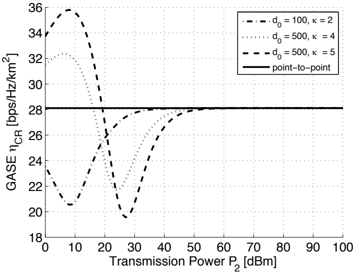


Figure: GASE

- When interfering transmitter is close to target receiver
  underlay cognitive transmission deteriorates both GASE and spectral efficiency.
- When interfering transmitter is far from target receiver
  - Different behavior in terms of spectral efficiency and GASE.
  - Both asymptotically approach to point-to-point link.

Conclusion

- Quantify spatial spectrum utilization efficiency of wireless systems.
- Characterize the spatial footprint of wireless transmission with affected area.
- Develop new performance metric for arbitrary wireless transmission.
- Capture the negative effect of radio power emission.
- On-going effort: GASE analysis for ad hoc wireless networks.