Stable, Efficient, and Accurate Marching Schemes for Solving Time Domain Integral Equations

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Outline

• Computational electromagnetics (CEM)
  ▪ Applications
  ▪ Motivation
  ▪ Overview of numerical methods

• Time Domain Integral Equations (TDIEs)
  ▪ A stable TDIE solver for high contrast dielectrics
  ▪ An explicit TDIE solver

• Real life examples
Electromagnetic Devices and Systems

Biomedical
- Magnetic, optical sensors
- Drug delivery systems
- Tissue-light interactions
- Imaging, MRI

Energy
- Solar cells
- Reservoir monitoring
- Subsurface imaging
- Low frequency antennas

Electromagnetic Devices and Systems
- Antenna systems
- On chip components
- Fast interconnect design
- Channel modeling
- Weather monitoring
- Antenna systems
- Wave propagation in buildings and cities

Communications and Computing

Environmental and Civil
Electromagnetic Devices and Systems

- Common challenges
  - Electrically large, many wavelengths long
  - Wide dynamic range of operation frequency
  - Geometrically intricate with dimensions varying by orders of magnitude
  - Many repetitive characterizations for design frameworks

- Basic understanding of electromagnetics and intuition lead the design

- Level of complexity calls for
  - Experimental tests/characterization
  - Numerical characterization/simulation

Captures all physical phenomena involved
Numerical Characterization

- Experimental design
  - Expensive and/or impossible tests
  - Repetitive processes, which oftentimes call for a start over, increases the cost
  - Expensive human labor
- Numerical characterization/simulation
  - Initial coding/programming cost
  - Cheaper repetitive processes on computers
  - Start overs are not too costly
  - Computing power getting cheaper
- CEM
  - Develops numerical tools
  - Enables scientific and technological advances in electromagnetics, optics, and photonics
Computational Electromagnetics (CEM)

- Two domains:
  - Frequency domain Solvers
  - Time domain Solvers
- Two types of methods:
  - Differential equation based solvers
  - Integral equation based solvers
**Frequency Domain Simulators**

- **Time-dependence:**
  \[ e^{j \omega t} \]

- **Time-derivative:**
  \[ \frac{\partial}{\partial t} \rightarrow j \omega \]

- **Time-integration:**
  \[ \int dt \rightarrow \frac{1}{j \omega} \]

- **Convolution → Multiplication**

- **Example Commercial Tools:**
  HFSS (FEM), Comsol (FEM), FISC (MOM), WIPL-D (MOM)

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**Frequency Domain Simulators:**

- **Positives**
  - Intuitively easier to understand
  - Easier to implement in general
  - Lower computational cost
  - Dispersion is easier to model

- **Negatives**
  - Strong nonlinearities cannot be modeled
  - Only single frequency results, no broadband data
  - Many simulations to obtain broadband results
  - Needs inverse Fourier transform as post-processors
Time Domain Simulators

- Time-dependence: Arbitrary (band limited)
- Fourier Transform to switch to frequency domain

- Positives
  - Strong nonlinearities can be modeled
  - Provides broadband data with a single simulation
  - Transient response is easy to get
  - Provides immediately the physics, no post-processing

- Negatives
  - More difficult to implement
  - Modeling dispersion requires computation of (costly) temporal convolutions
  - Higher computational cost

Example Commercial Tools:
Many available (FDTD), No well-known commercial tools (TD-FEM, MOT-TDIE)
Differential Equation Based Solvers

Time-dependent:

\[ \nabla \times \mathbf{E}(\mathbf{r}, t) = -\mu \frac{\partial \mathbf{H}(\mathbf{r}, t)}{\partial t} \]
\[ \nabla \times \mathbf{H}(\mathbf{r}, t) = \varepsilon \frac{\partial \mathbf{E}(\mathbf{r}, t)}{\partial t} + \mathbf{J}(\mathbf{r}, t) \]
\[ \nabla \cdot \mathbf{E}(\mathbf{r}, t) = \frac{\rho(\mathbf{r}, t)}{\varepsilon} \]
\[ \nabla \cdot \mathbf{H}(\mathbf{r}, t) = 0 \]

Time-harmonic:

\[ \frac{\partial}{\partial t} \rightarrow j\omega \]

Differential Equation Based Solvers:

- **Mechanics**
  - Discretize differential form of Maxwell equations
  - Approximate derivatives using neighboring elements (in time and space)

Example Commercial Tools:
HFSS (FEM), Comsol (FEM),
Many available (FDTD)

Typically preferred for problems of scatterers inhomogeneous background (example: photonic devices, waveguide problems)
Differential Equation Based Solvers

- **Time-dependent:**
  \[ \nabla \times \mathbf{E}(\mathbf{r}, t) = -\mu \frac{\partial \mathbf{H}(\mathbf{r}, t)}{\partial t} \]
  \[ \nabla \times \mathbf{H}(\mathbf{r}, t) = \varepsilon \frac{\partial \mathbf{E}(\mathbf{r}, t)}{\partial t} + \mathbf{J}(\mathbf{r}, t) \]
  \[ \nabla \cdot \mathbf{E}(\mathbf{r}, t) = \frac{\rho(\mathbf{r}, t)}{\varepsilon} \]
  \[ \nabla \cdot \mathbf{H}(\mathbf{r}, t) = 0 \]

- **Time-harmonic:**
  \[ \frac{\partial}{\partial t} \rightarrow j\omega \]

- **Example Commercial Tools:**
  HFSS (FEM), Comsol (FEM), Many available (FDTD)

- **Differential Equation Based Solvers:**
  - **Positives**
    - Straightforward to implement
    - Extension to inhomogeneous media is trivial
  - **Negatives/Challenges**
    - Numerical dispersion
    - Truncation of the (open) computation domain
    - Discretization of the computation domain
    - Typically time step size is constrained by the spatial discretization
    - Inaccurate geometry representation (FDTD)
Integral Equation Based Solvers:

- **Mechanics**
  - Replace scatterers with equivalent surface and volume currents
  - Find fields due to these currents using Green function
  - Apply boundary conditions to solve for unknowns

**Boundary Equation:**
\[
\partial_t E^{\text{inc}}(\mathbf{r}, t) \bigg|_{\text{tan}} = \partial_t E^{\text{sca}}(\mathbf{r}, t) \bigg|_{\text{tan}} \quad \mathbf{r} \in S
\]

Time-harmonic: \( \frac{\partial}{\partial t} \rightarrow j\omega \)

Example Commercial Tools: FISC (MOM), WPIL-D (MOM), Not available in time domain
Integral Equation Based Solvers:

- Positives
  - No phase dispersion
  - No grid truncation (exact radiation condition)
  - Time step size is not necessarily constrained by spatial discretization
  - Only the surface (or volume) of the object is discretized
  - Accurate representation of the geometry
  - Easy to hybridize with other time-domain methods

Example Commercial Tools:
FISC (MOM), WPIL-D (MOM),
Not available in time domain
Integral Equation Based Solvers:

- **Negatives/Challenges**
  - Require matrix inversion
  - More difficult to implement
  - Higher computational costs
  - Ill-conditioned in the presence of low frequency excitations
  - Ill-conditioned in the presence of multiscale geometries

Boundary Equation:
\[
\partial_t E^{inc}(r,t)_{\text{tan}} = \partial_t E^{sca}(r,t)_{\text{tan}} \quad r \in S
\]

Time-harmonic:
\[
\frac{\partial}{\partial t} \rightarrow j\omega
\]

Example Commercial Tools:
FISC (MOM), WPIL-D (MOM), Not available in time domain
CEM research group at KAUST develops novel frequency and time domain integral equation (FD/TDIE) solvers for characterizing electromagnetic wave interactions on electrically large and multi-scale structures and applies them in problems of electromagnetics and photonics. More specifically:

- Explicit and non-uniform, yet stable, time marching techniques for efficiently solving TDIEs
  - address the increase in computation time due to matrix inversion and large number of unknowns
- Mixed space and time discretization schemes for TDIEs
  - increased accuracy
- Exact boundary conditions in the form of TDIEs for terminating differential equation solvers
  - increased accuracy and efficiency
- Hybridization schemes between different-scale solvers in time domain
  - address the ill-conditioning due to multi-scale discretizations
- Apply them to problems of electromagnetics and photonics
• Time Domain Integral Equations (TDIEs)
  - A stable TDIE solver for high contrast dielectrics
  - An explicit TDIE solver
Formulation: TDVIE

- Volumetric scatter with $\varepsilon(r)$ and $\mu_0$ residing in free space with $\varepsilon_0$ and $\mu_0$
- Total volume: $V$
- Excitation: $E^{\text{inc}}(r,t)$ band-limited to $f_{\text{max}}$
- Current induced in $V$: $J(r,t)$

- Scattered field in terms of potentials

$$E^{\text{scat}}(r,t) = \int_V \frac{\mu_0 \partial_t J(r,t - R/c_0)}{4\pi R} \, d\mathbf{r}'$$

$$-\nabla \int_V \int_0^{t-R/c_0} \frac{\nabla' \cdot J(r',t')}{4\pi\varepsilon_0 R} \, d\mathbf{r}' \quad R = |\mathbf{r} - \mathbf{r}'|$$

- Equivalent current in terms of electric flux density

$$J(r,t) = \kappa(r) \partial_t D(r,t)$$

$$\kappa(r) = 1 - \frac{\varepsilon_0}{\varepsilon(r)}$$

- Electric flux density in terms of electric field intensity

$$E(r,t) = \varepsilon(r) D(r,t)$$
Formulation: TDVIE

\[ \partial_t \mathbf{E}(\mathbf{r}, t) = \partial_t \mathbf{E}(\mathbf{r}, t) - \partial_t \mathbf{E}^{\text{sca}}(\mathbf{r}, t) \]

- Fields satisfy

- Inserting everything above and enforcing the resulting equation at \( \mathbf{r} \in V \) yields the TDIVE in \( \mathbf{D}(\mathbf{r}, t) \)

\[ \partial_t \mathbf{E}^{\text{inc}}(\mathbf{r}, t) = \partial_t \mathbf{D}(\mathbf{r}, t)/\varepsilon(\mathbf{r}) \]

\[ + \int_V \frac{\mu_0 \kappa(\mathbf{r}') \partial_t^2 \mathbf{D}(\mathbf{r}, t - \mathbf{R}/c_0)}{4\pi R} d\mathbf{r}' \]

\[ - \nabla \int_V \frac{\nabla' \left[ \kappa(\mathbf{r}') \mathbf{D}(\mathbf{r}', t - \mathbf{R}/c_0) \right]}{4\pi \varepsilon_0 R} d\mathbf{r}' \]

- Volumetric scatter with \( \varepsilon(\mathbf{r}) \) and \( \mu_0 \) residing in free space with \( \varepsilon_0 \) and \( \mu_0 \)

- Total volume: \( V \)

- Excitation: \( \mathbf{E}^{\text{inc}}(\mathbf{r}, t) \) band-limited to \( f_{\text{max}} \)

- Current induced in \( V : \mathbf{J}(\mathbf{r}, t) \)
To numerically solve the TDVIE

Volume $V$ is divided into tetrahedrons

$\mathbf{D}(\mathbf{r},t)$ is expanded as

$$\mathbf{D}(\mathbf{r},t) \equiv \sum_{k'=1}^{N} \sum_{l'=0}^{N_t} I_{k',l'} T_{l'}(t) \mathbf{f}_{k'}(\mathbf{r})$$

Unknowns: $I_{k',l'}$

Temporal basis functions: $T_{l'}(t)$

Spatial basis functions: $\mathbf{f}_{k'}(\mathbf{r})$

$$\mathbf{f}_{k'}(\mathbf{r}) = \begin{cases} \pm \frac{A_k}{3V_k^\pm} (\mathbf{r} - \mathbf{r}_{k'}^\pm), & \mathbf{r} \in V_{k'}^\pm \\ 0, & \text{elsewhere} \end{cases}$$
Formulation: Temporal Basis Function Selection

- Lagrange interpolation function (LIF)
  - Wide spectrum (possible source of instability)
  - Discontinuous derivatives (possible source of instability)
- Approximate prolate spheroidal wave functions (APSWF)
  - Band-limited and short temporal support
  - Have continuous derivatives
  - Non-causal
- Non-causality is fixed by temporal extrapolation!
- Future values are predicted from past values
Formulation: MOT Solution

- Testing with $f_k(r)$ at times $l\Delta t$ yields

$$
\begin{bmatrix}
Z_0 & Z_{-1} & Z_{-2} \\
Z_1 & Z_0 & Z_{-1} & Z_{-2} \\
Z_2 & Z_1 & Z_0 & Z_{-1} & Z_{-2} \\
\vdots & \vdots & \vdots & \ddots & \ddots \\
Z_{N_g} & Z_{N_g-1} & \cdots & Z_2 & Z_1 & Z_0 & Z_{-1} & Z_{-2} \\
0 & Z_{N_g} & Z_{N_g-1} & \cdots & Z_2 & Z_1 & Z_0 & Z_{-1} & Z_{-2} \\
\vdots & \vdots & \vdots & \cdots & \cdots & \cdots & \cdots & \ddots & \ddots \\
0 & \cdots & 0 & Z_{N_g} & Z_{N_g-1} & \cdots & Z_2 & Z_1 & Z_0 \\
\end{bmatrix}
\begin{bmatrix}
I_1 \\
I_2 \\
I_3 \\
I_4 \\
\vdots \\
I_{N_t-1} \\
I_{N_t} \\
\end{bmatrix}
=
\begin{bmatrix}
V_1 \\
V_2 \\
V_3 \\
V_4 \\
\vdots \\
V_{N_t-1} \\
V_{N_t} \\
\end{bmatrix}
$$

- There are only $O(N^2)$ nonzero entries
- The matrix structure is due to
  - Causality
  - Time invariance of the Green function
  - Uniform time-step size

$$
N_g = \left\lfloor \frac{(D_{\text{max}} / c_0) / \Delta t + N^T}{N_t} \right\rfloor
$$
Formulation: MOT Solution

• Testing with $f_k(r)$ at times $l\Delta t$ yields

$$Z_0 I_l = V_l - \sum_{l'=1}^{l-1} Z_{l-l'} I_{l'} - \sum_{l'=l+1}^{l+N_l^T-1} Z_{l-l'} I_{l'}$$

\[
\{Z_{l-l'}\}_{k,k'} = \int_{V_k} \frac{1}{\varepsilon_k(r)} f_k(r) \cdot f_{k'}(r) T_{l'}(l\Delta t) \, dr + \frac{\mu_0}{4\pi} \left\langle f_k(r), \kappa_{k'}(r) f_k(r), \partial_t^2 T_{l'}(t) \right\rangle_{t=l\Delta t} \\
+ \frac{1}{4\pi\varepsilon_0} \left\langle \nabla \cdot f_k(r), \nabla \cdot [\kappa_{k'}(r) f_{k'}(r)], T_{l'}(t) \right\rangle_{t=l\Delta t}
\]

• How is this solved?

Solve for $I_1$

$$Z_0 I_1 = V_1 - Z_{-1} I_2 + Z_{-2} I_3$$

$$\hat{Z}_0 I_1 = V_1$$

Solve for $I_2$

$$Z_0 I_2 = V_2 - Z_{1} I_1 - Z_{-1} I_3 - Z_{-2} I_4$$

$$\hat{Z}_0 I_2 = V_2 - \hat{Z}_1 I_1$$

Solve for $I_3$

$$Z_0 I_3 = V_3 - Z_{1} I_2 - Z_{2} I_1 - Z_{-1} I_4 - Z_{-2} I_5$$

$$\hat{Z}_0 I_3 = V_3 - \hat{Z}_1 I_2 - \hat{Z}_2 I_1$$

....

Extrapolation
Formulation: Decaying and Oscillatory Modes

- Resonance modes of unit sphere

**TE mode**

\[
\frac{J_{n-1/2}(\beta \rho)}{J_{n+1/2}(\beta \rho)} = \frac{H_{n-1/2}^{(2)}(\beta \rho / \sqrt{\varepsilon_r})}{H_{n+1/2}^{(2)}(\beta \rho / \sqrt{\varepsilon_r})}
\]

**TM mode**

\[
\frac{n - J_{n-1/2}(\beta \rho)}{\beta \rho J_{n+1/2}(\beta \rho)} = \frac{n\varepsilon_r}{\beta \rho} + \sqrt{\varepsilon_r} \frac{H_{n-1/2}^{(2)}(\beta \rho / \sqrt{\varepsilon_r})}{H_{n+1/2}^{(2)}(\beta \rho / \sqrt{\varepsilon_r})}
\]

---

\[\varepsilon_r = 3\]

\[\varepsilon_r = 6\]

\[\varepsilon_r = 12\]
• Frequency sampling should be done on the complex frequency plane
• How to define a temporal extrapolation?

• Solution is expanded in terms of exponentials:
  \[ \phi(t) \sim \sum_{v=1}^{N_v} \alpha_v e^{\lambda_v t} \]

  \( \lambda_v \): complex numbers \( (v = 1 \ldots N_v) \)

  \( \alpha_v \): weighting coefficients

• Suppose that \( \lambda_v \) are known!

• Extrapolation coefficients:
  \[ \phi(t_j) = \sum_{l=1}^{k} \{p\}_l \phi(t_{j-1+l-k}) \]

• Matrix relation: \( \mathbf{A}_p \mathbf{p} = \mathbf{b} \)
  \[ \{\mathbf{A}_p\}_v,l = e^{\lambda_v t_l}; \quad v = 1, \ldots, N_v; \]
  \[ l = 1, \ldots, k \]

  \[ \{\mathbf{b}\}_v = e^{\lambda_v t_{k+1}} \]

• Solution is found by minimum norm least square solution

---

Formulation: Temporal Extrapolation

$$\varphi(t) = \sum_{i=1}^{N_V} \alpha_i e^{\lambda_i t}$$

$$Q = \begin{bmatrix} e^{\lambda_{1f_1}} & e^{\lambda_{2f_1}} & \cdots & e^{\lambda_{Nf_1}} \\
 e^{\lambda_{1f_2}} & e^{\lambda_{2f_2}} & \cdots & e^{\lambda_{Nf_2}} \\
 \vdots & \vdots & \ddots & \vdots \\
 e^{\lambda_{1f_M}} & e^{\lambda_{2f_M}} & \cdots & e^{\lambda_{Nf_M}} \end{bmatrix}$$

Maximum modulus principle

RRQR on \( Q \)

All steps are fully error controllable

---

Numerical Examples: Accuracy of Extrapolation

- **Signal:** Convolution of \( r(t) = \exp(-\vartheta t) \cos(2\pi \zeta t) \) with \( G(t) = \cos(2\pi f_0 t) \exp[-t^2 / (2\sigma^2)] \)
- **\( r(t) \)** resonance mode of unit sphere with \( \varepsilon_r = 12 \)

\[
f_0 = 34 \text{ MHz}, \quad f_{bw} = 17 \text{ MHz}, \quad \sigma = 3 / (2\pi f_{bw})
\]

\( \vartheta = 11.53 \text{ Np/ns}, \quad \zeta = 41.20 \text{ MHz} \)
Numerical Examples: Stability of the MOT Solution

- Scatterer: Dielectric unit sphere with increasing $\varepsilon_r$
- Eigenvalues of the MOT matrix for different temporal basis functions are compared

\[
\varepsilon_r = 3 \\
\varepsilon_r = 6 \\
\varepsilon_r = 12
\]
Numerical Examples: Accuracy of the MOT Solution

• Scatterer: Dielectric shell
• $\varepsilon_r = 3$, inner radius: 0.75m, outer radius 1m
• Excitation: $E^{\text{inc}}(\mathbf{r}, t) = \hat{\mathbf{p}} G(t - t_p - \mathbf{r} \cdot \hat{\mathbf{k}} / c)$

$$G(t) = \cos(2\pi f_0 t) \exp[-t^2 / (2\sigma^2)]$$

$$f_0 = 40\text{MHz}, f_{bw} = 20\text{MHz}, \sigma = 3 / (2\pi f_{bw}), t_p = 14\sigma$$
Numerical Examples: Accuracy of the MOT Solution

- Scatterer: Dielectric shell
- $\varepsilon_r = 100$, inner radius: 0.75m, outer radius 1m
- Excitation: $E^{\text{inc}}(r,t) = \hat{p}G(t - t_p - r \cdot \hat{k} / c)$

$$G(t) = \cos(2\pi f_0 t) \exp[-t^2 / (2\sigma^2)]$$

$f_0 = 18\text{MHz}$, $f_{bw} = 9\text{MHz}$, $\sigma = 3 / (2\pi f_{bw})$, $t_p = 14\sigma$
Formulation: Explicit MOT Solution

\[ Z_0 I_l = V_l - \sum_{l'-1}^{l-1} Z_{l-l'} I_{l'} \]  Implicit: \( Z_0 \) becomes full for large \( \Delta t \)

- Time derivative of the TDIE

\[ \frac{\partial_t D(r,t)}{\varepsilon(r)} = \partial_t E^{inc}(r,t) - \frac{\mu_0}{4\pi} \int_V dv' \kappa(r') \frac{\partial^3 D(r',\tau)}{R} + \frac{1}{4\pi\varepsilon_0} \nabla \int_V dv' \nabla' \cdot \kappa(r') \partial_t D(r',t') \]

- After discretization

\[ G \partial_t I_j = Z_0 I_j + V_j + \sum_{i=0}^{j-1} Z_{j-i} I_i, \quad G_{m,n} = \int_V \frac{\kappa_m(r)}{\varepsilon_n(r)} f_m(r) \cdot f_n(r) dv \]

- Relates time derivative of samples to samples
- Integrated in time using predictor-corrector methods – PE(CE)^m
- Enhanced using successive over relaxation (SOR)
At every time step $j$

Step 1: Compute
$$\mathbf{V}_{j}^{\text{fixed}} = \mathbf{V}_{j} + \mathbf{\tilde{V}}_{j}^{\text{sca}} = \mathbf{V}_{j} + \sum_{i=0}^{j-1} \mathbf{Z}_{j-i} \mathbf{I}_{i}$$

Step 2: Predict
$$\mathbf{I}_{j}^{(0)} = \sum_{l=1}^{k} \left[ \{ \mathbf{p} \}_l \mathbf{I}_{j-1+l-k} + \{ \mathbf{p} \}_{k+l} \partial_{t} \mathbf{I}_{j-1+l-k} \right]$$

Step 3: Evaluate $\partial_{t} \mathbf{I}_{j}$ by solving
$$\mathbf{G} \partial_{t} \mathbf{I}_{j} = \mathbf{V}_{j}^{\text{fixed}} + \mathbf{Z}_{0} \mathbf{I}_{j}^{(0)}$$

Step 4: Iterate until convergence in $\mathbf{I}_{j}^{(\nu)}, \nu = 1:m$
- Step 4.1: Correct
$$\mathbf{I}_{j}^{(\nu)} = \sum_{l=1}^{k} \left[ \{ \mathbf{c} \}_l \mathbf{I}_{j-1+l-k} + \{ \mathbf{c} \}_{k+l} \partial_{t} \mathbf{I}_{j-1+l-k} \right] + \{ \mathbf{c} \}_{2k+1} \partial_{t} \mathbf{I}_{j}^{(\nu-1)}$$
- Step 4.2: Apply SOR
$$\mathbf{I}_{j}^{(\nu)} = \alpha \mathbf{I}_{j}^{(\nu)} + (1 - \alpha) \mathbf{I}_{j}^{(\nu-1)}$$
- Step 4.3: Evaluate $\partial_{t} \mathbf{I}_{j}^{(\nu)}$ by solving
$$\mathbf{G} \partial_{t} \mathbf{I}_{j}^{(\nu)} = \mathbf{V}_{j}^{\text{fixed}} + \mathbf{Z}_{0} \mathbf{I}_{j}^{(\nu)}$$

$$\mathbf{G}_{m,n} = \int \mathbf{K}_{m}(\mathbf{r}) \frac{\epsilon_{m}(\mathbf{r})}{\epsilon_{n}(\mathbf{r})} \mathbf{f}_{m}(\mathbf{r}) \cdot \mathbf{f}_{n}(\mathbf{r}) d\mathbf{r}$$

Gram matrix is well-conditioned and sparse regardless of time step size.
Formulation: Explicit MOT Solution

**Explicit:**

\[
C^{\text{exp}} \sim 2kN + m(2k+1)N + mN + (m+1)(\gamma N) + (m+1)N^G_{\text{iter}}F_{\text{iter}}(7N)
\]

\[
\begin{align*}
\text{Z}_0I_j & \quad \text{Iterative solution of } G \\
\end{align*}
\]

\[
C^{\text{exp}} \sim mN^G_{\text{iter}}F_{\text{iter}}N
\]

**Implicit:**

\[
C^{\text{imp}} \sim N^\text{imp}_{\text{iter}}F_{\text{iter}}\gamma N
\]

**Formulae:**

- \(k\): length of coeff.
- \(m\): no. of corr. steps
- \(\gamma\): sparseness factor
- \(N_s\): no. of unknowns
- \(F_{\text{iter}}\): complexity of iter.
- \(N^\text{imp}_{\text{iter}}\): implicit iter.
- \(N^G_{\text{iter}}\): Gram matrix iter.

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<th>Implicit</th>
<th>Explicit</th>
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<td><strong>High freq.</strong></td>
<td>( C^{\text{imp}} \sim N^\text{imp}<em>{\text{iter}}F</em>{\text{iter}}N )</td>
<td>( C^{\text{exp}} \sim mN^G_{\text{iter}}F_{\text{iter}}N )</td>
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<tr>
<td>( \gamma \ll N )</td>
<td>( C^{\text{imp}} \sim N^\text{imp}<em>{\text{iter}}F</em>{\text{iter}}N )</td>
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<td>( C^{\text{imp}} \sim N^\text{imp}<em>{\text{iter}}F</em>{\text{iter}}N^2 )</td>
<td>( C^{\text{exp}} \sim mN^2 )</td>
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<td>( m \ll N^\text{imp}<em>{\text{iter}}F</em>{\text{iter}} )</td>
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Numerical Examples: Excitation

- **Excitation:**

\[ E^{\text{inc}}(\mathbf{r},t) = \hat{p}_{\text{pol}} \cos \left[ 2\pi f_0 (t - t_p - \mathbf{r} \cdot \hat{k} / c) \right] e^{-\frac{(t-t_p-r\cdot\hat{k}/c)^2}{2\sigma^2}} \]

\[ f_0 \quad : \text{Center frequency} \]

\[ f_{\text{bw}} \quad : \text{Effective bandwidth} \]

\[ \hat{p}_{\text{pol}} \quad : \text{Polarization} \quad (\hat{p}_{\text{pol}} = \hat{x}) \]

\[ \hat{k} \quad : \text{Direction of propagation} \]

\[ \sigma = \frac{3}{2\pi f_{\text{bw}}} \quad t_p = 6\sigma \]

- **Predictor-corrector coefficients:**

Number of steps : \( k = 22 \)

Convergence cond. : \( \varepsilon = 10^{-16} \)
Numerical Examples: Convergence of SOR

- Effect of SOR on PE(CE)^m scheme

\[ N = 9364 \quad f_0 = 25 \text{ MHz} \quad f_{bw} = 12.5 \text{ MHz} \]
\[ \Delta t = 0.5 \text{ ns} \quad \epsilon_r = 2.25 \epsilon_0 \quad \alpha = 0.7 \]

![Graph showing convergence of SOR](image)

\[ E^{inc}(r,t) \]
• Accuracy

\[ N = 9364 \quad f_0 = 25 \text{ MHz} \quad f_{bw} = 12.5 \text{ MHz} \]
\[ \Delta t = 0.5 \text{ ns} \quad \varepsilon_r = 2.25\varepsilon_0 \quad \alpha = 0.7 \]
• Application Examples

  ▪ Biomedical
    - Blood cell-light interaction
    - Thin film detection

  ▪ Computing/Communications
    - Computer board
    - Antennas on complex platforms

  ▪ Environmental and Civil
    - Wave propagation in a building
Blood Cell – Light Interaction

- Used in biomedical applications: devices utilizing lasers for disease diagnosis
- Can provide essential information for blood related diseases

![Blood Cell Image]

\[ \epsilon_b = 1.81 \]

\[ \epsilon(\mathbf{r}') = 1.97 \]

Normalised intensity of the electric field distribution at three consecutive moments in time:

- \( t = 86\text{fs} \)
- \( t = 96\text{fs} \)
- \( t = 106\text{fs} \)

The amplitude of the transient electric field induced at the centre

\[ N = 1,031,550 \]

\[ N_t = 2666 \]
The layered microsphere acts as a lensing device to produce
- a focused and localized light beam in space
- a narrow high intensity in space
The scattered field is analysed for the identification of sub-wave length defects

Detection of sub-wavelength dielectric nano-features using a narrow high intensity light beam - Photonic Nano-Jet

\[ N = 592,226 \]

\[ N_t = 2280 \]

Photonic Nano-Jet emerging from the shadow side of a layered dielectric microsphere
Detection of sub-wavelength dielectric nano-features using a narrow high intensity light beam - Photonic Nano-Jet

Photonic Nano-Jet emerging from the shadow side of a layered dielectric microsphere
Computer Board
Computer Board
LPMA on an Aircraft

Log-Periodic Monopole Array (LPMA)

LPMA feed-network

Monopole 1

Monopole 8

\[ a = 0.29 \]

0.5 m
LPMA on an Aircraft

- 34 MHz
- 61 MHz
- 70 MHz
- 88 MHz
LPMA on an Aircraft

- Proposed Simulator
- Unaccelerated Simulator

- Proposed Simulator
- Frequency-domain Simulator

- $L_{\text{m},1}$
- $L_{\text{m},2}$
- $L_{\text{m},3}$

- $L_{\text{m},4}$
- $L_{\text{m},5}$
- $L_{\text{m},6}$

- $L_{\text{m},7}$
- $L_{\text{m},8}$
- $L_{\text{m},9}$

- $L_{\text{m},10}$

- Monopole 1
- Monopole $i$
- Monopole $s$

- 50 $\Omega$
Antennas on a Car

Antennas for:
Collision Avoidance, GPS, Cellular, AM, FM, …
Antennas on a Car

$V_s(t), V_{\text{max}} = 1 \text{ V}$

$f \to 800 \text{ MHz}$

$m400 \text{ MHz}$

Monopoles 1 and 2

Monopole 3

Coupled Voltage on node 1
Retrodirective Antenna Array on a Car

Operating Frequency: 5 GHz
Retrodirective Antenna Array on a Car

Radiation Patterns at 5 GHz
Communication Antenna Array on a Car

Operating Frequency:
3.5 GHz
Communication Antenna Array on a Car

Radiation Patterns at 3.5 GHz
Wave Propagation in a Building

\[ f_{\text{max}} = 1.25 \text{ GHz} \]
\[ N_s = 930486 \]
\[ N_t = 800 \]
\[ \Delta t = 76 \text{ ps} \]
\[ c\Delta t \approx 2.28 \text{ mm} \]
PhD Students
- Ismail Uysal (Sep. 2011-May 2015): “Quantum-corrected time domain surface integral equation solvers for plasmonics”
- Sadeed B. Sayed (Sep. 2012-May 2016): “Highly stable time domain volume integral equations for dielectric scatterers”

Post-Doctoral Researchers
- Yifei Shi (May 2012-May 2015)
- Mohamed Farhat (Sep. 2012-Sep. 2015)
- Ping Li (Sep. 2014-Sep 2015)

Alumni
- Muhammad Amin (PhD degree, Sep. 2010-May 2014): Assistant Professor, Electrical Engineering, Sarhad University of Science and Information Technology (SUIT), Peshawar, Pakistan
- Abdulla Desmal (MS degree, Sep. 2009-Dec. 2010): PhD student at KAUST
- Umair Khalid (MS degree, Dec. 2010): MBA Student, Strategy and Finance, Emory University, Atlanta, GA.
- Kostyantyn Sirenko (Post-Doc) (February 2010-August 2014)