Performance Analysis of Cognitive Multihop Relaying with M-QAM Detect-and-Forward in Nakagami-m Fading Channels

Mustapha Benjillali[1], Amal Hyadi[2], Daniel Benevides da Costa[3], and Mohamed-Slim Alouini[2]
[1]Communications Systems Department, INPT, Rabat, Morocco
[2]Computer, Electrical, and Mathematical Science and Engineering (CEMSE) Division,
King Abdullah University of Science and Technology (KAUST), Thuwal, Saudi Arabia
[3]Wireless Telecommunications Research Group (GTEL), Federal University of Ceará (UFC), CE, Brazil
Emails: benjillali@ieee.org, amal.hyadi@kaust.edu.sa, danielbcosta@ieee.org, slim.alouini@kaust.edu.sa

***Abstract*—In this work, we investigate the performance of cognitive multihop regenerative relaying systems in the "underlay" spectrum sharing scenario. The multiple relays perform "detect-and-forward" relaying strategy to convey a message with an order M quadrature amplitude modulation (M-QAM) from the source to the destination over independent but not necessarily identical Nakagami-m fading channels. We adopt a closed-form analysis framework based on univariate and bivariate Meijer G-functions to derive the end-to-end error performance (in terms of bit and symbol error rates), the outage probability, and the ergodic capacity. Various numerical examples are presented to illustrate the results with a large combination of system and fading parameters, and simulation results confirm the accuracy of our closed-form analysis.**

I. Introduction

Nowadays, *relaying* and *cooperation* [1] are well-established approaches to mitigate the effects of wireless mobile channels in order to increase the capacity and/or the coverage of wireless communications systems.

Among the many forms of cooperation, "multihop" relaying systems (where the source communicates with the destination through a number of relays) have practical advantages [2] like the flexibility of infrastructure, areas of possible deployment, and connectivity. In addition, they allow for coverage increase with lower transmit powers.

Numerous relaying schemes were presented and analyzed in the literature, but practical considerations (such as complexity, memory requirements, and delay) make Detect-and-Forward (DetF) [3] [4] an interesting relaying strategy in resource-limited networks (e.g., sensor networks) because the relay only demodulates—instead of decoding—the received signals and retransmits the detected binary words to the destination.

On another level, *cognitive radio* [5] is an enabling physical layer technology for dynamic spectrum access. Efficient use of the radio spectrum is possible by allowing unlicensed/secondary users (SUs) to exploit portions of the spectrum initially allocated to the licensed/primary users (PUs). This spectrum access is granted provided that the PU communication is not affected, which is possible by controlling the transmission powers of the SUs with respect to an interference threshold or *temperature* at the PU. The coexistence of PUs and SUs in the same spectrum band is known as "spectrum sharing" [6]; the "underlay" transmission scheme is a common example of such a coexistence.

Finally, the combination of cognitive techniques with cooperation and relaying strategies yields what is usually referred to as *cooperative spectrum sharing systems* (CSSS). In this work, we focus on the multihop detect-and-forward CSSS.

The end-to-end bit error rate (BER) of conventional multihop regenerative relaying over fading channels was analyzed in [7], [8] and, more recently, in [9]. In [10], the authors analyzed the "uncoded" error performance and the outage probability of a regenerative Decode-and-Forward multihop cognitive relaying system. However, the analysis did not take into consideration the realistic transmit power limitation at the cognitive nodes, and the expressions were derived assuming only Rayleigh fading channels. In [11], considering the interference constraint on the primary user in an underlay cognitive context, the outage probability of a dual-hop communication was derived in Nakagami-m fading channels.

In this work, we extend and generalize the analysis to the context of multihop cognitive communications with the underlay spectrum sharing strategy. We derive closed-form expressions for many end-to-end performance metrics of the secondary network communication over Nakagami-m fading channels, namely: the outage probability, the average BER and the average symbol error rate (SER) when an arbitrary M-ary quadrature amplitude modulation (M-QAM) is adopted, and the ergodic capacity.

II. System Model

A. Cognitive Network Setup

We consider a multihop cognitive secondary network operating in the underlay spectrum sharing context with the presence of a primary network. The secondary network consists of one source (r_0), one destination (r_N), and $N-1$ intermediate DetF

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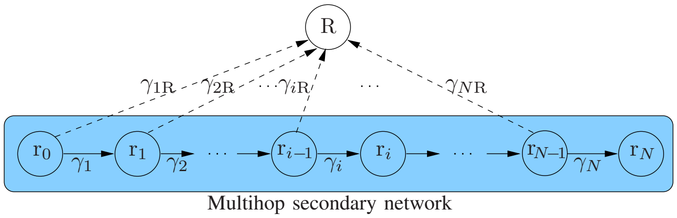

Fig. 1. System model with a N-hop cognitive relaying network and one primary receiver.

relays (r_i, $i = 1, \ldots, N-1$), as shown in Fig. 1. A total of N hops is thus used to convey the information from the source to the destination. We assume that all nodes are equipped with a single antenna and that they all operate in the half-duplex mode. The primary network is represented by a primary receiver R with a maximum interference constraint $I_{\max}$ (i.e., the maximum acceptable interference temperature from the secondary transmitters). However, the analysis could be directly extended to the case where multiple primary receivers exist; the equivalent interference constraint in this scenario is simply the minimum of individual interference constraints among all primary receivers. Primary transmitters are not taken into consideration since the focus of this work is on the secondary network communication, and we can assume for example that the primary transmitters are located far enough from the secondary network that their interference with the secondary receivers is negligible.

Channel coefficients $h_{\mathrm{r}_{i-1}\mathrm{r}_i}$ on the i-th relaying hop (H_i), and $h_{\mathrm{r}_{i-1}\mathrm{R}}$ on the i-th interference link (I_i), are from slow independent and not necessarily identically distributed Nakagami-m fading profiles with respective fading parameters m_{H_i}, m_{I_i}. Hence, $|h_{\mathrm{r}_{i-1}\mathrm{r}_i}|^2$ and $|h_{\mathrm{r}_{i-1}\mathrm{R}}|^2$ follow Gamma distributions with shape parameters m_{H_i} and m_{I_i} and scale parameters $\overline{\mu}_{\mathrm{H}_i}$ and $\overline{\mu}_{\mathrm{I}_i}$, respectively. All channel coefficients are assumed to be perfectly known at the receiving node, as channel estimation issues are out of scope here. Along our analysis, the fading parameters m_{H_i} and m_{I_i} are assumed to be integers, and the noise over all channels is zero-mean additive white Gaussian (AWGN) with the same variance N_0.

The source node and all cognitive relay nodes transmit the initial message and the detected binary words in intermediate hops using the same rectangular M-QAM. The adopted modulation order is denoted by $m = \log_2(M)$, not to be confused with the fading parameters m_{H_i} and m_{I_i}.

A maximum transmit power constraint P is imposed on transmit nodes, at the source and the relays, and the average transmit signal-to-noise ratio (SNR) is given by $\mathsf{SNR} = P/N_0$. Hence, the instantaneous SNR at the output of the i-th hop (i.e., the channel between nodes r_{i-1} and r_i), $i = 1, \ldots, N$, is given by

$$\gamma_i = \min\left\{\frac{I_{\max}}{|h_{\mathrm{r}_{i-1}\mathrm{R}}|^2}, P\right\} \cdot \frac{|h_{\mathrm{r}_{i-1}\mathrm{r}_i}|^2}{N_0}. \quad (1)$$

In order to alleviate the expressions, and without any loss of generality, we set $N_0 = 1$ in the rest of the analysis.

B. Statistics of the Per-Hop SNR

1) Expression of the Cumulative Distribution Function (CDF): From (1), and after a few derivations, the CDF of the instantaneous SNR on the i-th hop is given by [11, Eq. (14)]

$$\begin{aligned} F_{\gamma_i}(\gamma_i) &= \Gamma_{\mathrm{reg}}\left(m_{\mathrm{I}_i}, \frac{m_{\mathrm{I}_i} I_{\max}}{\overline{\mu}_{\mathrm{I}_i} P}\right) \\ &+ \gamma_{\mathrm{reg}}\left(m_{\mathrm{I}_i}, \frac{m_{\mathrm{I}_i} I_{\max}}{\overline{\mu}_{\mathrm{I}_i} P}\right) \cdot \gamma_{\mathrm{reg}}\left(m_{\mathrm{H}_i}, \frac{m_{\mathrm{H}_i} \gamma_i}{\overline{\mu}_{\mathrm{H}_i} P}\right) \\ &- \sum_{k=0}^{m_y-1} \frac{\Phi_{\mathrm{I}_i}}{k! \beta_i(\gamma_i)^{\alpha_{ik}+1}} \left(\frac{m_{\mathrm{H}_i}\gamma_i}{\overline{\mu}_{\mathrm{H}_i} I_{\max}}\right)^k \Gamma\left(\alpha_{ik}+1, \beta_i(\gamma_i)\frac{I_{\max}}{P}\right), \end{aligned} \quad (2)$$

where $\Gamma(\cdot,\cdot)$ and $\Gamma_{\mathrm{reg}}(\cdot,\cdot)$ are respectively the upper and regularized upper incomplete gamma functions [12, Eq. (8.350.2)], $\gamma_{\mathrm{reg}}(\cdot,\cdot)$ is the regularized lower incomplete gamma function [12, Eq. (8.350.1)], and

$$\Phi_{\mathrm{I}_i} = \frac{m_{\mathrm{I}_i}^{m_{\mathrm{I}_i}-1}}{\overline{\mu}_{\mathrm{I}_i}}, \quad \alpha_{ik} = m_{\mathrm{I}_i} + k - 1, \quad \beta_i(x) = \frac{m_{\mathrm{I}_i}}{\overline{\mu}_{\mathrm{I}_i}} + \frac{m_{\mathrm{H}_i}}{\overline{\mu}_{\mathrm{H}_i}} \cdot \frac{x}{I_{\max}}.$$

2) Derivation of the Probability Density Function (PDF): By calculating the derivative of the CDF expression in (2) with respect to γ_i, we obtain a closed-form expression of the PDF of the SNR on the i-th hop as

$$\begin{aligned} f_{\gamma_i}(\gamma_i) &= \gamma_{\mathrm{reg}}\left(m_{\mathrm{I}_i}, \frac{m_{\mathrm{I}_i} I_{\max}}{\overline{\mu}_{\mathrm{I}_i} P}\right) \cdot \frac{\gamma_i^{m_{\mathrm{H}_i}-1} \mathrm{e}^{-\gamma_i/(\overline{\mu}_{\mathrm{H}_i} P)}}{\Gamma(m_{\mathrm{H}_i}) \left(\overline{\mu}_{\mathrm{H}_i} P\right)^{m_{\mathrm{H}_i}}} \\ &+ \sum_{k=0}^{m_{\mathrm{H}_i}-1} \frac{\Phi_x}{k! \beta_i(\gamma_i)^{\alpha_{ik}+1}} \left(\frac{m_{\mathrm{H}_i}}{\overline{\mu}_{\mathrm{H}_i} I_{\max}}\right)^k \\ &\times \left\{ \frac{m_{\mathrm{H}_i}}{\overline{\mu}_{\mathrm{H}_i} P} \gamma_i^k \left(\beta_i(\gamma_i)\frac{I_{\max}}{P}\right)^{\alpha_{ik}} \exp\left(-\beta_i(\gamma_i)\frac{I_{\max}}{P}\right) \right. \\ &\left. + \left(\frac{m_{\mathrm{H}_i}}{\overline{\mu}_{\mathrm{H}_i} I_{\max}} \frac{\alpha_{ik}+1}{\beta_i(\gamma_i)} \gamma_i^k - k\gamma_i^{k-1}\right) \Gamma\left(\alpha_{ik}+1, \beta_i(\gamma_i)\frac{I_{\max}}{P}\right) \right\}, \end{aligned} \quad (3)$$

where $\Gamma(\cdot)$ is the gamma function [12, Eq. (8.310.1)].

III. Closed-Form Performance Analysis

A. End-to-End Outage Probability

Here, we generalize the expression of the outage probability over a single link [11] to obtain the total end-to-end outage probability in the multihop scenario. In the adopted network setup, we can write

$$\begin{aligned} \mathsf{P}_{\mathsf{out}}(\gamma_{\mathrm{th}}) &= \sum_{k=1}^{N+1} \Pr[\gamma_k < \gamma_{\mathrm{th}}] \cdot \prod_{i=1}^{k-1} \Pr[\gamma_i \geq \gamma_{\mathrm{th}}] \\ &= \sum_{k=1}^{N+1} F_{\gamma_k}(\gamma_{\mathrm{th}}) \cdot \prod_{i=1}^{k-1} \left[1 - F_{\gamma_i}(\gamma_{\mathrm{th}})\right], \end{aligned} \quad (4)$$

which is obtained in closed-form with the help of (2).

B. End-to-End Average BER

For the sake of notational brevity, we adopt the following notations in the rest of the paper

$$a_i=\frac{m_{\mathrm{H}_i}}{\overline{\mu}_{\mathrm{H}_i}P},\qquad b_i=\frac{m_{\mathrm{I}_i}}{\overline{\mu}_{\mathrm{I}_i}},\qquad \text{and}\qquad c_i=\frac{m_{\mathrm{H}_i}}{\overline{\mu}_{\mathrm{H}_i}I_{\max}},$$

$$A_{1i}=\Gamma_{\mathrm{reg}}\left(m_{\mathrm{I}_i},\frac{m_{\mathrm{I}_i}I_{\max}}{\overline{\mu}_{\mathrm{I}_i}P}\right),\ A_{3il}=\frac{\Gamma(m_{\mathrm{I}_i}+l)}{\Gamma(m_{\mathrm{I}_i})\,l!}\left(\frac{m_{\mathrm{I}_i}}{\overline{\mu}_{\mathrm{I}_i}}\right)^{m_{\mathrm{I}_i}}\left(\frac{m_{\mathrm{H}_i}}{\overline{\mu}_{\mathrm{H}_i}}\right)^{l}.$$

The end-to-end average BER of a multihop relaying system is given by [7], [8]

$$\overline{\mathsf{BER}}_{\mathsf{e2e}}=\sum_{i=1}^{N}\overline{\mathsf{BER}}_i\prod_{j=i+1}^{N}\left(1-2\,\overline{\mathsf{BER}}_j\right),\tag{5}$$

where $\overline{\mathsf{BER}}_i$ denotes the average BER over the i-th hop, and is given by

$$\overline{\mathsf{BER}}_i=\int_0^\infty \mathsf{BER}(\gamma)\cdot f_{\gamma_i}(\gamma)\,d\gamma,\tag{6}$$

where $f_{\gamma_i}(\gamma)$ is the PDF of the instantaneous SNR on the i-th link given by (3), and BER is the instantaneous BER of an M-QAM transmission over an AWGN channel given by [13]

$$\mathsf{BER}(\gamma)=\frac{1}{\sqrt{M}\log_2\sqrt{M}}\sum_{k=1}^{\log_2\sqrt{M}}\sum_{n=0}^{\nu_k}\Phi_{nk}\,\mathrm{erfc}(\sqrt{\omega_n\gamma}),\tag{7}$$

with $\nu_k=\left(1-2^{-k}\right)\sqrt{M}-1$, $\omega_n=\dfrac{3\,(2n+1)^2\log_2 M}{2M-2}$, and $\Phi_{nk}=(-1)^{\left\lfloor\frac{n2^{k-1}}{\sqrt{M}}\right\rfloor}\left(2^{k-1}-\left\lfloor\dfrac{n2^{k-1}}{\sqrt{M}}+\dfrac{1}{2}\right\rfloor\right)$. The average single-hop BER can thus be expressed as

$$\overline{\mathsf{BER}}_i=\frac{2}{\sqrt{M}\log_2\sqrt{M}}\sum_{k=1}^{\log_2\sqrt{M}}\sum_{n=0}^{\nu_k}\Phi_{nk}\mathcal{J}_{ni},\tag{8}$$

where, taking into consideration the CDF expression in (2),

$$\begin{aligned}\mathcal{J}_{ni}&=\int_0^\infty \mathrm{Q}\left(\sqrt{2\omega_n\gamma}\right)f_{\gamma_i}(\gamma)\,d\gamma\\&=\frac{1}{2}\sqrt{\frac{\omega_n}{\pi}}\int_0^\infty\gamma^{-1/2}\,\mathrm{e}^{-\omega_n\gamma}\,F_{\gamma_i}(\gamma)\,d\gamma\\&=\frac{1}{2}\sqrt{\frac{\omega_n}{\pi}}\left[A_{1i}\cdot\mathcal{I}_1+(1-A_{1i})\cdot\mathcal{I}_{2i}-\sum_{l=0}^{m_{\mathrm{H}_i}-1}A_{3il}\cdot\mathcal{I}_{3il}\right],\end{aligned}\tag{9}$$

with

$$\mathcal{I}_1=\int_0^\infty\gamma^{-1/2}\,\mathrm{e}^{-\omega_n\gamma}\,d\gamma=\frac{\Gamma(1/2)}{\omega_n^{1/2}}=\sqrt{\frac{\pi}{\omega_n}},\tag{10}$$

$$\begin{aligned}\mathcal{I}_{2i}&=\int_0^\infty\gamma^{-1/2}\,\mathrm{e}^{-\omega_n\gamma}\,\gamma_{\mathrm{reg}}(m_{\mathrm{H}_i},a_i\gamma)\;d\gamma\\&=\sqrt{\frac{\pi}{\omega_n}}-\sum_{j=0}^{m_{\mathrm{H}_i}-1}\frac{a_i^j}{j!}\,\frac{\Gamma(j+1/2)}{(\omega_n+a_i)^{j+1/2}}\end{aligned}\tag{11}$$

using the finite series form $\gamma_{\mathrm{reg}}(s,z)=1-\mathrm{e}^{-z}\sum_{p=0}^{s-1}\frac{z^p}{p!}$, and

$$\begin{aligned}\mathcal{I}_{3il}&=\int_0^\infty\gamma^{l-1/2}\,\mathrm{e}^{-\omega_n\gamma}\;\frac{\Gamma(\alpha_{il}+1,\beta_i(\gamma)I_{\max}/P)}{\beta_i(\gamma)^{\alpha_{il}+1}}\,d\gamma\\&=\sum_{j=0}^{\alpha_{il}}\frac{(I_{\max}/P)^j}{j!\,\Gamma(\alpha_{il}-j+1)}\left(\frac{c_i}{b_i}\right)^{-l+1/2}\frac{\mathrm{e}^{-b_i\,I_{\max}/P}}{b_i^{\alpha_{il}-j+1}}\\&\quad\times\mathrm{G}_{1,2}^{2,1}\left(\frac{b_i}{c_i}\left(\omega_n+c_i\frac{I_{\max}}{P}\right)\,\middle|\,\begin{matrix}1/2-l\\0,m_{\mathrm{I}_i}-j-1/2\end{matrix}\right),\end{aligned}\tag{12}$$

where $\mathrm{G}_{:,:}^{:,:}(\cdot|\cdot)$ is the Meijer G-function [12, Eq. (9.301)] that is readily available in the standard mathematical packages such as MATHEMATICA, MATLAB and MAPLE. Note that (12) could also be expressed in terms of the confluent hypergeometric function [12, Eq. (9.211.1)].

Finally, the expressions in (9), (8), and consequently the end-to-end average BER in (5), are obtained in closed-form.

C. End-to-End Average SER

Adopting the same analysis approach in [14], we can get the exact closed-form expression for the average SER of the analyzed system. The problem reduces to averaging a product of two Q-functions over the fading distribution [15] of the i-th hop, i.e.,

$$\begin{aligned}\mathcal{I}_{\mathsf{SER}_i}&=\int_0^\infty\mathrm{Q}(A\sqrt{\gamma})\,\mathrm{Q}(B\sqrt{\gamma})\,f_{\gamma_i}(\gamma)\,d\gamma\\&=\frac{A}{\sqrt{8\pi}}\mathcal{I}_{A,B}+\frac{B}{\sqrt{8\pi}}\mathcal{I}_{B,A},\end{aligned}\tag{13}$$

where $f_{\gamma_i}(\gamma)$ is given by (3), yielding

$$\begin{aligned}\mathcal{I}_{X,Y}&=\int_0^\infty\gamma^{-1/2}\mathrm{e}^{-\gamma X^2/2}\mathrm{Q}(Y\sqrt{\gamma})\,F_{\gamma_i}(\gamma)\,d\gamma\\&=A_{1i}\cdot\mathcal{I}_1'+(1-A_{1i})\cdot\mathcal{I}_{2i}'-\sum_{l=0}^{m_{\mathrm{H}_i}-1}A_{3il}\cdot\mathcal{I}_{3il}',\end{aligned}\tag{14}$$

where A_{1i} and A_{3il} are defined in subsection III-B, and $\mathcal{I}_1'$ and $\mathcal{I}_{2i}'$ can be obtained with the help of the Meijer G-function representation as

$$\begin{aligned}\mathcal{I}_1'&=\int_0^\infty\gamma^{-1/2}\mathrm{e}^{-\gamma X^2/2}\mathrm{Q}(Y\sqrt{\gamma})\,d\gamma\\&=\frac{1}{X\sqrt{2\pi}}\cdot\mathrm{G}_{2,2}^{2,1}\left(\frac{Y^2}{X^2}\,\middle|\,\begin{matrix}1/2,1\\0,1/2\end{matrix}\right),\end{aligned}\tag{15}$$

$$\begin{aligned}\mathcal{I}_{2i}'&=\int_0^\infty\gamma^{-1/2}\mathrm{e}^{-\gamma X^2/2}\gamma_{\mathrm{reg}}(m_{\mathrm{H}_i},a_i\gamma)\cdot\mathrm{Q}(Y\sqrt{\gamma})\,d\gamma\\&=\frac{1}{X\sqrt{2\pi}}\cdot\mathrm{G}_{2,2}^{2,1}\left(\frac{Y^2}{X^2}\,\middle|\,\begin{matrix}1/2,1\\0,1/2\end{matrix}\right)-\sum_{j=0}^{m_{\mathrm{H}_i}-1}\frac{a_i^j}{j!}\left(\frac{X^2}{2}+a_i\right)^{-j-1/2}\\&\quad\times\mathrm{G}_{2,2}^{2,1}\left(\frac{Y^2}{X^2+2a_i}\,\middle|\,\begin{matrix}1/2-j,1\\0,1/2\end{matrix}\right),\end{aligned}\tag{16}$$

and

$$\mathcal{I}_{3il}'=\int_0^\infty\gamma^{-1/2}\mathrm{e}^{-\gamma X^2/2}\frac{\Gamma_{\mathrm{reg}}\left(\alpha_{il}+1,\beta_i(\gamma)\dfrac{I_{\max}}{P}\right)}{\beta_i(\gamma)^{\alpha_{il}+1}}\mathrm{Q}(Y\sqrt{\gamma})d\gamma$$

$$= e^{-b_i I_{\max}/P} \sum_{j=0}^{\alpha_{il}} \frac{(I_{\max}/P)^j (-1)^{\alpha_{il}-j+1}}{j!\,(\alpha_{il}-j)! \cdot b_i c_i^{\alpha_{il}-j+1}} \left(\frac{Y^2}{2}\right)^{m_{\mathrm{I}_i}-j-1/2}$$

$$\times \mathrm{G}_{2,1:0,1:3,3}^{0,2:1,0:1,3}\left(\Psi_1, \Psi_2 \left| \begin{matrix} m_{\mathrm{I}_i}+\frac{1}{2}-j, m_{\mathrm{I}_i}-j \\ m_{\mathrm{I}_i}-\frac{1}{2}-j \end{matrix} \right| \begin{matrix} - \\ 0 \end{matrix} \left| \begin{matrix} 0,1,1 \\ 1,1,\alpha_{il}-j \end{matrix} \right.\right), \quad (17)$$

where $\mathrm{G}^{\cdot,\cdot:\cdot,\cdot:\cdot,\cdot}_{\cdot,\cdot:\cdot,\cdot:\cdot,\cdot}(\cdot,\cdot|\cdot|\cdot|\cdot)$ is the bivariate Meijer G-function [16] for which an implementation code in MATHEMATICA is proposed in [17], $\Psi_1 = \dfrac{X^2P + 2c_i I_{\max}}{Y^2 P}$, and $\Psi_2 = \dfrac{2c_i}{b_i Y^2}$.

D. Ergodic Capacity

The end-to-end ergodic capacity of a multihop relaying system with DetF can be written as

$$\overline{\mathsf{C}}_{e2e} = \min_{i=1,\ldots,N} \overline{\mathsf{C}}_i, \quad (18)$$

where

$$\overline{\mathsf{C}}_i = \int_0^\infty \log_2(1+\gamma)\, f_{\gamma_i}(\gamma)\, d\gamma \quad (19)$$

is the ergodic capacity of the i-th hop. Taking into consideration the expression of the PDF of the instantaneous SNR presented in (3), this capacity (19) can be written as

$$\overline{\mathsf{C}}_i = (1-A_{1i}) \frac{a_i^{m_{\mathrm{H}_i}}}{\Gamma(m_{\mathrm{H}_i})} \cdot \mathcal{I}''_{1i} + \frac{b^{m_{\mathrm{I}_i}}}{\Gamma(m_{\mathrm{I}_i})} \sum_{l=0}^{m_{\mathrm{H}_i}-1} \frac{a_i^l P^l}{l!\, I_{\max}^l}$$

$$\times \left[a_i \left(\frac{I_{\max}}{P}\right)^{\alpha_{il}} \cdot \mathcal{I}''_{2il} + \frac{a_i(\alpha_{il}+1)P}{I_{\max}} \cdot \mathcal{I}''_{3il} - l \cdot \mathcal{I}''_{3i(l-1)}\right], \quad (20)$$

where a_i, b_i, c_i and A_{1i} are defined in subsection III-B, and

$$\mathcal{I}''_{1i} = \int_0^\infty \log_2(1+\gamma)\, \gamma^{m_{\mathrm{H}_i}-1} \exp(-a_i\gamma)\, d\gamma$$

$$= \frac{a_i^{-m_{\mathrm{H}_i}}}{\ln(2)} \cdot \mathrm{G}_{3,2}^{1,3}\left(\frac{1}{a_i} \left| \begin{matrix} 1-m_{\mathrm{H}_i}, 1, 1 \\ 1, 0 \end{matrix} \right.\right), \quad (21)$$

$$\mathcal{I}''_{2il} = \int_0^\infty \log_2(1+\gamma) \frac{\gamma^k}{\beta_i(\gamma)} \exp\left(\frac{-\beta_i(\gamma) I_{max}}{P}\right) d\gamma$$

$$= \frac{e^{-b_i I_{\max}/P}}{\ln(2) c_i} \left(\frac{c_i I_{\max}}{P}\right)^{-l}$$

$$\times \mathrm{G}_{1,0:2,2:2,2}^{0,1:1,2:1,2}\left(\frac{P/c_i}{I_{\max}}, \frac{P/b_i}{I_{\max}} \left| \begin{matrix} 1-l \\ - \end{matrix} \right| \begin{matrix} 1,1 \\ 1,0 \end{matrix} \left| \begin{matrix} 1,1 \\ 1,1 \end{matrix} \right.\right), \quad (22)$$

$$\mathcal{I}''_{3il} = \int_0^\infty \frac{\gamma^l \log_2(1+\gamma)}{\beta_i(\gamma)^{\alpha_{il}+2}} \Gamma\left(\alpha_{il}+1, \beta_i(\gamma)\frac{I_{max}}{P}\right) d\gamma$$

$$= \frac{e^{-b_i I_{\max}/P} (-1)^{\alpha_{il}+2}}{\ln(2)(m_{\mathrm{I}_i}+l)!\, b_i c_i^{m_{\mathrm{I}_i}+l}} \left(\frac{c_i I_{\max}}{P}\right)^{m_{\mathrm{I}_i}-1} \sum_{j=0}^{\alpha_{il}} \binom{\alpha_{il}}{l} \left(\frac{b_i I_{\max}}{P}\right)^{\alpha_{il}-j}$$

$$\times \mathrm{G}_{2,1:2,2:3,3}^{0,2:1,2:1,3}\left(\frac{P/c_i}{I_{\max}}, \frac{P/b_i}{I_{\max}} \left| \begin{matrix} m_{\mathrm{I}_i}-j-1, m_{\mathrm{I}_i} \\ m_{\mathrm{I}_i}-1 \end{matrix} \right| \begin{matrix} 1,1 \\ 1,0 \end{matrix} \left| \begin{matrix} 0,1,1 \\ 1,1,m_{\mathrm{I}_i}+l \end{matrix} \right.\right). \quad (23)$$

IV. Numerical Examples and Discussion

In this section, we discuss representative numerical examples to illustrate the accuracy of our analysis, and show its potential applications in the design of cognitive multihop relaying systems.

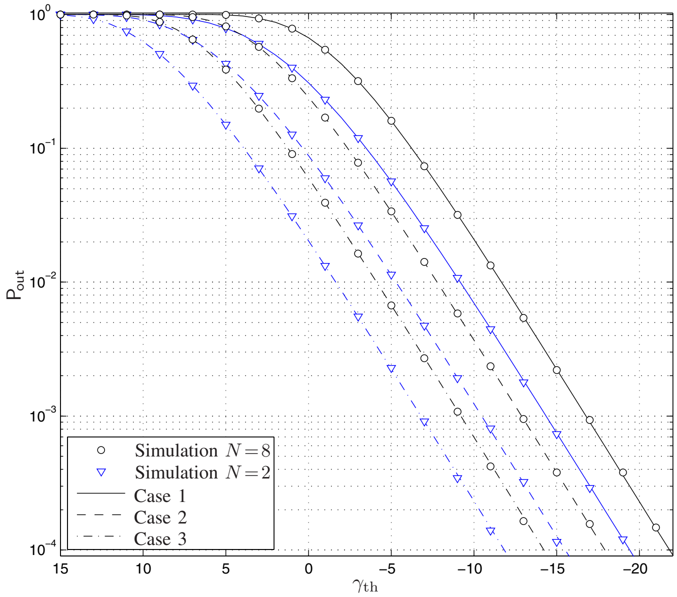

Fig. 2. End-to-end outage probability after 2 and 8 hops, lines correspond to analytical results and simulation results are represented with markers.

In Fig. 2, we show the end-to-end outage probability of the analyzed multihop setup with $N = 2$ and $N = 8$. All links have identical parameters, i.e., $m_{\mathrm{I}_i} = 1$, $m_{\mathrm{H}_i} = 2$, $\overline{\mu}_{\mathrm{I}_i} = 2$ dB and $\overline{\mu}_{\mathrm{I}_i} = 4$ dB. Three combination scenarios of $I_{\max}$ and P are illustrated: a) Case 1 with $I_{\max} = 4$ dB and $P = 6$ dB, b) Case 2 with $I_{\max} = 8$ dB and $P = 8$ dB, and c) Case 3 with $I_{\max} = 12$ dB and $P = 10$ dB. The figure shows that the diversity order of the analyzed scheme is 2, independently from the number of hops. However, a performance degradation around 4 dB is observed when the number of hops is increased from 2 to 8 for example.

In Fig. 3, we illustrate the end-to-end BER after $N = 4$ hops with 4-QAM (i.e., QPSK) and 16-QAM. Simulation results, represented with markers only, confirm the accuracy of our closed-form derivations. The results correspond to the case of non identically fading links with $m_I = [2, 1, 1, 2]$ and $m_H = [3, 1, 2, 1]$. From the figure, we can see that the uncoded error performance of the system is considerably limited by the interference temperature at the primary receiver (i.e., presence of high error floors).

In Fig. 4, we illustrate the end-to-end SER after $N = 2$ and $N = 4$ hops with 4-QAM and 16-QAM, and for two interference constraints at the primary receiver $I_{\max} = 15$ dB and $I_{\max} = 20$ dB. Note that, although theses interference temperatures are relatively high, their impact on the end-to-end performance is considerable. In addition, increasing the number of hops (from 2 to 4) degrades considerably the error performance of the system (with an initial gap of almost 3 dB).

Finally, the end-to-end ergodic capacity after $N = 2$ and $N = 4$ hops is represented in Fig. 5 for three different interference temperatures at the primary receiver. We can see that the impact of the interference limitation at the primary receiver is practically independent from the number of hops.

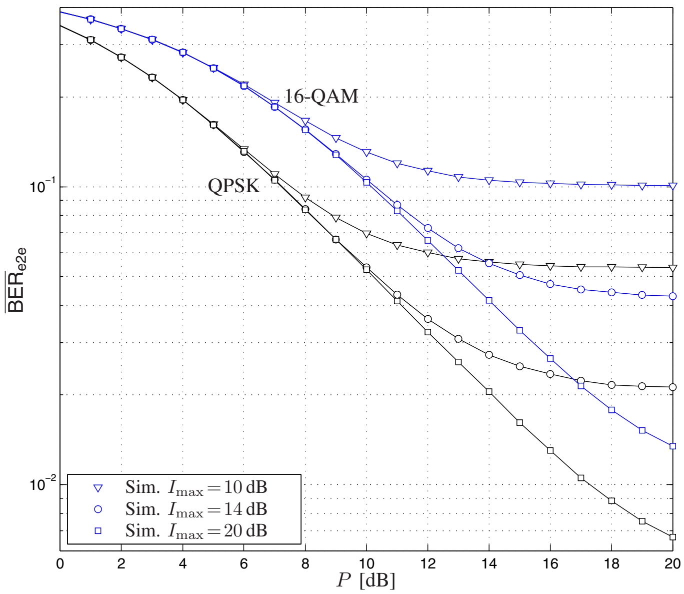

Fig. 3. End-to-end BER after 4 hops with QPSK and 16-QAM for different interference temperatures. Solid lines correspond to analytical results, and simulation results are represented with markers only.

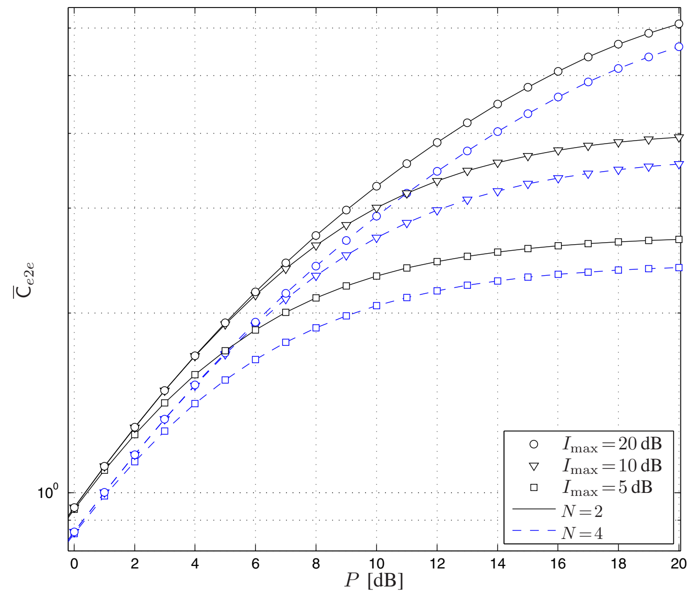

Fig. 5. End-to-end ergodic capacity after 2 and 4 hops for different interference temperatures.

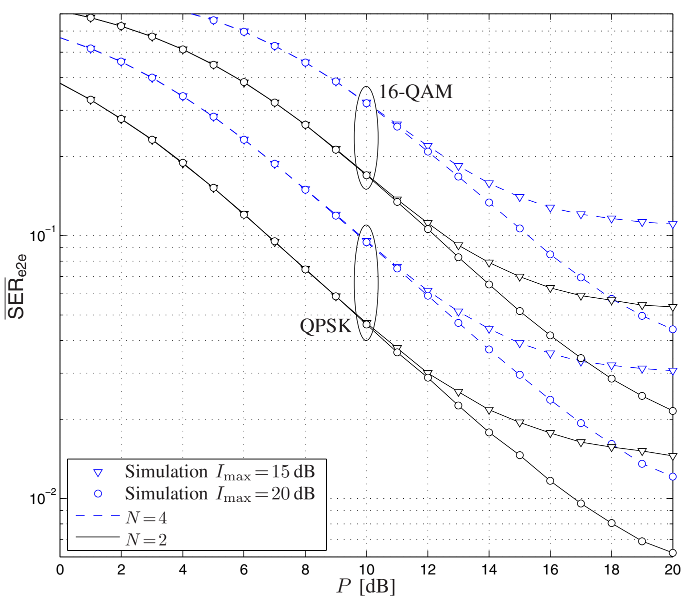

Fig. 4. End-to-end SER after 2 and 4 hops with QPSK and 16-QAM for different interference temperatures.

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