Overlay Cognitive Radio Systems with Adaptive Two-Way Relaying

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***Abstract*—In this paper, we propose a spectrum sharing mechanism with a two-phase two-way relaying protocol for an overlay cognitive network. The system comprises two primary users (PUs) and two secondary users (SUs). One of the SUs acts as a relay for the PUs and gains spectrum sharing as long as he respects outage probability constraints of the primary system. Moreover, we consider that the relaying node performs an optimal power allocation scheme that minimizes the outage performance of the secondary receiver. Closed form expressions for the outage probability are derived for the cases of Decode-and-Forward (DF), Amplify-and-Forward (AF), and adaptive relaying. Numerical simulations are presented to illustrate and compare the obtained results.**

I. Introduction

The world of communications has witnessed, in the last years, an increasing demand for wireless applications. As a result of this growth, the spectrum is becoming overcrowded. Cognitive radio [1] is a brilliant solution that has been proposed to get over this handicap of spectrum scarcity. Interweave, underlay, or overlay [2] are three different approaches that cognitive radio systems could employ. In interweave approach, secondary users (SUs) should periodically sense the environment to detect spectrum holes. They are allowed to transmit only when primary users (PUs) are idle. Underlay approach allows, on the contrary, the coexistence of primary and secondary users but under the constraint that SUs cause limited interference to PUs. For overlay approach, in which we are interested in this work, SUs allocate partial power to transmit PUs' data and use the remaining power to send their own information. However, maintaining or even improving the signal to noise ratio (SNR) of PUs is a crucial constraint to be respected by the SUs in this case. An alternative constraint consists on limiting the outage probability of the primary network given a desired target [3].

Two-way relaying is a cooperative technique that has been proposed to ensure a better spectral efficiency [4]. Instead of requiring four time slots, as it is the case with the traditional one-way relaying, the communication will take only two time slots with a typical two-phase two-way relaying protocol. In the first phase, the Multiple Access Phase (MAP), the users send their information to the relay node in the same frequency band and the same time slot. In the second phase, the Broadcast Phase (BP), the relay broadcasts the overlapped information.

Combining cognitive radio and two-way relaying provides a propitious mean to enhance the spectrum utilization efficiency. Depending on the system design and the employed approach, the two-way communication could be performed whether in the primary or the secondary network.

This combining idea of spectrum sharing and two-way relaying was the essence of recent research works. The great majority of these works investigate the case of underlay cognitive systems with the two-way relaying performed at the secondary network [5]–[9]. Different performance analysis and power allocation techniques have been proposed for such systems. However, the number of works in which the overlay approach is incorporated with two-way relaying is still restricted. The authors in [10] examine the case of an overlay system with a three-phase two-way relaying protocol. The communication is performed in three stages: two stages to convey the data to the secondary relay followed by a broadcast stage. The DF relaying strategy was adopted and a power allocation scheme has been proposed. Another work in [11] aims to find an optimal power allocation scheme for the primary and secondary users subject to interference constraints. In the proposed system, the secondary users perform two-way relaying to assist the primary users communication and are allowed to communicate with the cognitive base station only when the primary users are idle.

In this work, we consider the case of an overlay cognitive network with the more spectrally efficient two-phase two-way relaying scheme. In the proposed system, the secondary user performs an adaptive relaying technique to convey the primary data and makes use of the broadcast phase to send its own information. An optimal power allocation at the relaying secondary node is investigated toward a minimum outage probability at the secondary receiver given outage constraints on the primary system.

The rest of the this paper is organized as follows. Section

This work was funded in part by an NPRP grant from Qatar National Research Fund (QNRF) (A member of Qatar Foundation).

II provides the details of the system model and formulates the optimization problem. Section III represents the optimal power allocation analysis in the cases of DF, AF and adaptive relaying. The numerical results are shown in Section IV. Finally, Section V concludes the whole paper.

II. System Model and Problem Formulation

We consider an overlay cognitive radio network with two primary users PU_1 and PU_2, and two secondary users SU_k and SU_j, as shown in Fig. 1. The primary users support relay functionality and employ a two-phase two-way relaying strategy whenever the channel drops below a certain threshold. In this case, the secondary user SU_k serves as a relay to assist the communication between the primary users. In exchange, SU_k can send its own data to SU_j in the broadcast phase of the relaying process, subject to constraints on the outage probabilities of the primary users.

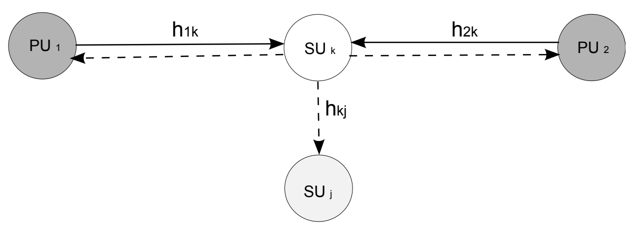

Fig. 1. The overlay cognitive system with two-way relaying.

We assume that the primary and secondary nodes use single transmit/receive antennas, that all nodes operate in the half duplex mode and that the channels are reciprocal. Let x_1, x_2 and x_s be the symbols transmitted by PU_1, PU_2 and SU_k respectively. We assume that $E\{|x_1|^2\} = E\{|x_2|^2\} = E\{|x_s|^2\} = 1$.

In the first phase of the two-way relaying process, both primary users send their symbols to SU_k. The received signal can be written as:

$$y_{\text{r}_k} = \sqrt{P_1}h_{1k}x_1 + \sqrt{P_2}h_{2k}x_2 + n_{\text{r}_k}, \tag{1}$$

where P_1 and P_2 are the transmit powers of PU_1 and PU_2 respectively, n_{r_k} is an Additive White Gaussian Noise (AWGN) such as $n_{\text{r}_k} \sim \mathcal{N}(0, \sigma^2)$ and $h_{ik}/i \in \{1, 2\}$ is the channel gain from PU_i to SU_k. We consider an aggregate channel coefficient. That is $h_{ik} = \sqrt{d_{ik}^{-\beta}}g_{ik}$ where β is the path loss coefficient, d_{ik} is the normalized distance between PU_i and SU_k and g_{ik} is the fading coefficient modeled as a Rayleigh variable. Thus, $|g_{ik}|^2$ is exponentially distributed, i.e., $|g_{ik}|^2 \sim \text{Exp}(\delta_{ik})$ and then $\gamma_{ik} = |h_{ik}|^2 \sim \text{Exp}(\lambda_{ik})$ with $\lambda_{ik} = d_{ik}^{\beta}\delta_{ik}$.

In the second phase, SU_k uses a fraction α of its power P_{r_k} to transmit the primary users symbols. The remaining power $(1-\alpha)P_{\text{r}_k}$ is used to transmit its own symbol x_s. The fraction α is obtained as a solution of an optimization problem that minimizes the outage probability of SU_j subject to ensuring a same or a better outage performances for the primary system as if no spectrum sharing is performed. The optimization problem could be expressed as:

$$\begin{aligned} &\underset{0<\alpha<1}{\text{Minimize}} && \text{P}_{\text{out}}^{\text{SU}_j} \\ &\text{subject to} && \begin{cases} \text{P}_{\text{out}}^{\text{PU}_1} \le \text{P}_{\text{out}_1} \\ \text{P}_{\text{out}}^{\text{PU}_2} \le \text{P}_{\text{out}_2}, \end{cases} \end{aligned} \tag{2}$$

where $\text{P}_{\text{out}}^{\text{SU}_k}$, $\text{P}_{\text{out}}^{\text{PU}_1}$ and $\text{P}_{\text{out}}^{\text{PU}_2}$ represent the outage probabilities of SU_k, PU_1 and PU_2 respectively. While P_{out_1} and P_{out_2} are the outage probabilities of PU_1 and PU_2 without spectrum sharing.

III. Optimal Power Allocation

In this section, we investigate the optimal power allocation at the secondary user. We seek, indeed, to find the optimal fraction of power used by SU_k to send the primary users data. This requires solving the optimization problem in (2). Before considering the adaptive relaying case, we study the cases of DF and AF.

A. DF Relaying

In the first phase of relaying, SU_k will try to decode the received primary users symbols x_1 and x_2. If he succeeded in decoding both symbols, then he will broadcast a bitwise XOR of the two symbols $x_1 \oplus x_2$ in the second phase. Otherwise an outage will be declared. According to [12], SU_k is able to decode x_1 and x_2 if $R_{1k} \ge R_1$, $R_{2k} \ge R_2$ and $R_{\sum} \ge R_1 + R_2$ with

$$R_{ik} = \frac{1}{2}\log\left(1 + \frac{P_i|h_{ik}|^2}{\sigma^2}\right), \quad i \in \{1, 2\} \tag{3}$$

$$R_{\sum} = \frac{1}{2}\log\left(1 + \frac{P_1|h_{1k}|^2}{\sigma^2} + \frac{P_2|h_{2k}|^2}{\sigma^2}\right). \tag{4}$$

In case the decoding is successful, the received signal at $\text{PU}_i/i \in \{1, 2\}$, in the broadcast phase, could be written as

$$y_i = \sqrt{\alpha P_{\text{r}_k}}h_{ik}(x_1 \oplus x_2) + \sqrt{(1-\alpha)P_{\text{r}_k}}h_{ik}x_s + n_i, \tag{5}$$

with n_i is an AWGN such as $n_i \sim \mathcal{N}(0, \sigma^2)$.

PU_i will decode the bitwise XOR signal and then perform self interference cancellation to eliminate its own symbol. An outage is declared if $R_{ki}^{DF} < R_i$ where

$$R_{ki}^{DF} = \frac{1}{2}\log\left(1 + \frac{\alpha P_{\text{r}_k}|h_{ik}|^2}{(1-\alpha)P_{\text{r}_k}|h_{ik}|^2 + \sigma^2}\right). \tag{6}$$

The outage probability of PU_i could be then written as

$$\begin{aligned} \text{P}_{\text{out}}^{\text{PU}_i} &= 1 - \text{P}[R_{1k} \ge R_1]\,\text{P}[R_{2k} \ge R_2] \\ &\times \text{P}\big[R_{\sum} \ge R_1 + R_2 | R_{1k} \ge R_1, R_{2k} \ge R_2\big]\,\text{P}\big[R_{ki}^{DF} \ge R_i\big], \end{aligned} \tag{7}$$

where

$$\begin{aligned} \text{P}[R_{ik} \ge R_i] &= \text{P}\left[\gamma_{ik} \ge \underbrace{\frac{\sigma^2(2^{2R_i}-1)}{P_i}}_{\gamma_i}\right] \\ &= \exp(-\lambda_{ik}\gamma_i), \end{aligned} \tag{8}$$

$$\mathrm{P}\left[R_{\sum} \geq R_1{+}R_2|R_{1k}{\geq}R_1, R_{2k}{\geq}R_2\right] = \tag{9}$$
$$\mathrm{P}\left[P_1\gamma_{1k}{+}P_2\gamma_{2k} \geq \underbrace{\sigma^2\left(2^{2(R_1+R_2)}{-}1\right)}_{\gamma_{\Sigma}}\Big|\gamma_{1k}{\geq}\gamma_1,\gamma_{2k}{\geq}\gamma_2\right],$$

We can see that $P_1\gamma_{1k} + P_2\gamma_{2k}$ is a sum of two weighted exponential variables. Using the cumulative distribution function derived in [13], the equation in (9) could be written as

$$\begin{aligned}
&\mathrm{P}\big[R_{\sum}{\geq}R_1{+}R_2|R_{1k}{\geq}R_1, R_{2k}{\geq}R_2\big] = \\
&1-\exp\!\Big(-\frac{\lambda_{1k}}{P_1}\gamma_1 - \frac{\lambda_{2k}}{P_2}\gamma_2\Big) \\
&+\Big\{\exp\!\Big(-\frac{\lambda_{2k}}{P_2}\gamma_2\Big) + \frac{\lambda_{1k}P_2}{\lambda_{2k}P_1{-}\lambda_{1k}P_2}\Big\}\exp\!\Big(-\frac{\lambda_{1k}}{P_1}\gamma_{\Sigma}\Big) \\
&-\frac{\lambda_{1k}P_2}{\lambda_{2k}P_1{-}\lambda_{1k}P_2}\exp\!\Big(\Big(\frac{\lambda_{2k}}{P_2}{-}\frac{\lambda_{1k}}{P_1}\Big)\gamma_1\Big)\exp\!\Big(-\frac{\lambda_{2k}}{P_2}\gamma_{\Sigma}\Big),
\end{aligned} \tag{10}$$

and for the probability that $R_{ki}^{DF} \geq R_i$, we have

$$\mathrm{P}\big[R_{ki}^{DF} \geq R_i\big] = \mathrm{P}\left[\left(1{-}\left(1{-}\alpha\right)2^{2R_i}\right)\gamma_{ik} \geq \underbrace{\frac{\sigma^2\left(2^{2R_i}{-}1\right)}{P_{\mathrm{r}_k}}}_{\overline{\gamma}_i}\right]$$

by taking $\alpha > 1 - 2^{-2R_i}$, we can write

$$\mathrm{P}\left[R_{ki}^{DF} \geq R_i\right] = \exp\left(-\frac{\lambda_{ik}\overline{\gamma}_i}{1-(1-\alpha)\,2^{2R_i}}\right), \tag{11}$$

otherwise (if $\alpha \leq 1 - 2^{-2R_i}$), $\mathrm{P}\left[R_{ki}^{DF} \geq R_i\right] = 0$.

By adopting the following notation

$$\begin{aligned}
C = \exp(-\lambda_{1k}\gamma_1{-}\lambda_{2k}\gamma_2)\Big[&1-\exp\!\Big(-\frac{\lambda_{1k}}{P_1}\gamma_1 - \frac{\lambda_{2k}}{P_2}\gamma_2\Big) \\
&+\Big\{\exp\!\Big(-\frac{\lambda_{2k}}{P_2}\gamma_2\Big) + \frac{\lambda_{1k}P_2}{\lambda_{2k}P_1{-}\lambda_{1k}P_2}\Big\}\exp\!\Big(-\frac{\lambda_{1k}}{P_1}\gamma_{\Sigma}\Big) \\
&-\frac{\lambda_{1k}P_2}{\lambda_{2k}P_1{-}\lambda_{1k}P_2}\exp\!\Big(\Big(\frac{\lambda_{2k}}{P_2}{-}\frac{\lambda_{1k}}{P_1}\Big)\gamma_1\Big)\exp\!\Big(-\frac{\lambda_{2k}}{P_2}\gamma_{\Sigma}\Big)\Big],
\end{aligned} \tag{12}$$

and substituting (8), (11) and (10) in (7) with $\alpha > \max_{i=1,2}\left(1{-}2^{-2R_i}\right)$, the outage probability of PU_i can be given by

$$\mathrm{P}_{\mathrm{out}}^{\mathrm{PU}_i} = 1 - C\exp\left(-\frac{\lambda_{ik}\overline{\gamma}_i}{1-(1-\alpha)\,2^{2R_i}}\right). \tag{13}$$

In order to increase his chances in decoding the data sent by SU_k, SU_j will try at first to decode the primary users symbols x_1 and x_2 using both signals received in the the two phases of communication and then he will decode the symbol x_s. We have indeed

$$\begin{aligned}
y_{\mathrm{r}_j} &\overset{\mathrm{Phase1}}{=} \sqrt{P_1}h_{1j}x_1 + \sqrt{P_2}h_{2j}x_2 + n_{\mathrm{r}_j} \\
y_{\mathrm{r}_j} &\overset{\mathrm{Phase2}}{=} \sqrt{\alpha P_{\mathrm{r}_k}}h_{kj}\left(x_1 \oplus x_2\right) + \sqrt{(1-\alpha)\,P_{\mathrm{r}_k}}h_{kj}x_s + n_{\mathrm{r}_j}
\end{aligned}$$

with $n_{\mathrm{r}_j} \sim \mathcal{N}(0,\sigma^2)$and $h_{ij}/i \in \{1,2\}$ is the channel gain from PU_i to SU_j. Writing the given signals in a matrix form, we get

$$\begin{aligned}
Y_{\mathrm{r}_j} &= \begin{bmatrix}\sqrt{P_1}h_{1j} & \sqrt{P_2}h_{2j} \\ \sqrt{\alpha P_{\mathrm{r}_k}}h_{kj} & \sqrt{\alpha P_{\mathrm{r}_k}}h_{kj}\end{bmatrix}\begin{bmatrix}x_1 \\ x_2\end{bmatrix} \\
&\quad + \begin{bmatrix}1 & 0 \\ 1 & \sqrt{(1-\alpha)\,P_{\mathrm{r}_k}}h_{kj}\end{bmatrix}\begin{bmatrix}n_{\mathrm{r}_j} \\ x_s\end{bmatrix}.
\end{aligned} \tag{14}$$

Using the V-BLAST algorithm with Zero Forcing or Minimum Mean Squared Error detection [14, pp. 124-131], SU_j can decode x_1 and x_2 with a negligible probability of error. Assuming perfect detection, SU_j will then try to decode x_s. Thus, the achievable rate for link $k \to j$ can be obtained as

$$R_{kj}^{DF} = \frac{1}{2}\log\left(1 + \frac{(1-\alpha)\,P_{\mathrm{r}_k}|h_{kj}|^2}{\sigma^2}\right). \tag{15}$$

An outage is declared if $R_{kj}^{DF} < R_{\mathrm{r}_j}$. The overall outage probability of SU_j is then given by

$$\begin{aligned}
\mathrm{P}_{\mathrm{out}}^{\mathrm{SU}_j} &= 1 - \mathrm{P}[R_{1k} \geq R_1]\,\mathrm{P}[R_{2k} \geq R_2] \\
&\times \mathrm{P}\big[R_{\sum}{\geq}R_1{+}R_2|R_{1k}{\geq}R_1, R_{2k}{\geq}R_2\big]\,\mathrm{P}\big[R_{kj}^{DF} \geq R_{\mathrm{r}_j}\big],
\end{aligned} \tag{16}$$

where

$$\begin{aligned}
\mathrm{P}\left[R_{kj}^{DF} \geq R_j\right] &= \mathrm{P}\left[(1-\alpha)\,\gamma_{kj} \geq \underbrace{\frac{\sigma^2\left(2^{2R_{\mathrm{r}_j}}-1\right)}{P_{\mathrm{r}_k}}}_{\gamma_{\mathrm{r}_j}}\right] \\
&= \exp\left(-\frac{\lambda_{kj}\gamma_{\mathrm{r}_j}}{1-\alpha}\right).
\end{aligned} \tag{17}$$

Substituting (8), (10) and (17) in (16), the outage probability of SU_j is given by

$$\mathrm{P}_{\mathrm{out}}^{\mathrm{SU}_j} = 1 - C\exp\!\left(-\frac{\lambda_{kj}\gamma_{\mathrm{r}_j}}{1-\alpha}\right), \tag{18}$$

where C is defined in (12).

The optimization problem in (2) can then be written for the DF case as

$$\begin{aligned}
&\underset{\alpha}{\text{Minimize}} \quad 1 - C\exp\left(-\frac{\lambda_{kj}\gamma_{\mathrm{r}_j}}{1-\alpha}\right) \\
&\text{subject to} \quad \begin{cases}1 - C\exp\left(-\frac{\lambda_{ik}\overline{\gamma}_i}{1-(1-\alpha)\,2^{2R_i}}\right) \leq \mathrm{P}_{\mathrm{out}_i} \\ \max_i\left(1-2^{-2R_i}\right) < \alpha < 1\end{cases}
\end{aligned} \tag{19}$$

with $i \in \{1,2\}$.

By solving the optimization problem in (19), the optimal fraction of power α_{opt} is given by

$$\alpha_{\mathrm{opt}} = \min\left(1, \max_{i=1,2}\left(1-2^{-2R_i}, \alpha_i\right)\right) \tag{20}$$

with $\alpha_i = 1 - \left(1 + \frac{\lambda_{ik}\overline{\gamma}_i}{\ln\left(\frac{1-\mathrm{P}_{\mathrm{out}_i}}{C}\right)}\right)2^{-2R_i}$.

B. AF Relaying

For the AF relaying case, SU_k will amplify the received signal (1) by $\sqrt{\alpha P_{\mathrm{r}_k}}$, add its own symbol x_s amplified by $\sqrt{(1-\alpha)P_{\mathrm{r}_k}}$ and then broadcast it. The received signal at $\mathrm{PU}_i / i \in \{1,2\}$ could be then written as

$$\begin{aligned} y_i &= \sqrt{\alpha P_{\mathrm{r}_k}} h_{ik} \left(\sqrt{P_1} h_{1k} x_1 + \sqrt{P_2} h_{2k} x_2 + n_{\mathrm{r}_k} \right) \\ &+ \sqrt{(1-\alpha) P_{\mathrm{r}_k}} h_{ik} x_s + n_i. \end{aligned} \tag{21}$$

After self interference cancellation at PU_i, an outage is declared if $R_{ki}^{AF} < R_i$ where

$$R_{ki}^{AF} = \frac{1}{2}\log\left(1 + \frac{\alpha P_{\mathrm{r}_k} |h_{1k}|^2 |h_{2k}|^2 P_l}{(1-\alpha) P_{\mathrm{r}_k} |h_{ik}|^2 + (\alpha P_{\mathrm{r}_k} |h_{ik}|^2 + 1)\sigma^2}\right)$$

with $l \in \{1,2\}$ and $l \neq i$.

The outage probability of PU_i is given by

$$\begin{aligned} \mathrm{P}_{\text{out}}^{\mathrm{PU}_i} &= \mathrm{P}\left[R_{ki}^{AF} < R_i\right] \qquad (22) \\ &= \mathrm{P}\left[\frac{\gamma_{1k}\gamma_{2k}}{\gamma_{ik} + \frac{\sigma^2}{P_{\mathrm{r}_k}(1-\alpha(1-\sigma^2))}} < \frac{2^{2R_i}-1}{\alpha P_l \left(1-\alpha(1-\sigma^2)\right)^{-1}} \right]. \end{aligned}$$

Using the outage probability expression for non-regenerative systems, derived in [15], the outage probability of PU_i could be then written as

$$\begin{aligned} \mathrm{P}_{\text{out}}^{\mathrm{PU}_i} &= 1 - 2\sqrt{\frac{\sigma^2(2^{2R_i}-1)}{\alpha P_l P_{\mathrm{r}_k} \lambda_{1k}\lambda_{2k}}} K_1\left(2\sqrt{\frac{\sigma^2(2^{2R_i}-1)}{\alpha P_l P_{\mathrm{r}_k} \lambda_{1k}\lambda_{2k}}}\right) \\ &\times \exp\left(-\frac{2^{2R_i}-1}{\alpha P_l \lambda_{lk} \left(1-\alpha(1-\sigma^2)\right)^{-1}}\right), \end{aligned} \tag{23}$$

where $K_1(.)$ is the first-order modified Bessel function of the second kind defined in [16, eq. (3.324.1)].

For his part, SU_j will adopt the same decoding process as in the DF relaying case. However, the received signal at SU_j, in the broadcast phase, is different this time. For the AF relaying case, the matrix form of the two received signals, presented in (14) for DF, is given by

$$\begin{aligned} Y_{\mathrm{r}_j} &= \begin{bmatrix} \sqrt{P_1} h_{1j} & \sqrt{P_2} h_{2j} \\ \sqrt{\alpha P_{\mathrm{r}_k} P_1} h_{kj} h_{1k} & \sqrt{\alpha P_{\mathrm{r}_k} P_2} h_{kj} h_{2k} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \\ &+ \begin{bmatrix} 1 & 0 & 0 \\ 1 & \sqrt{\alpha P_{\mathrm{r}_k}} h_{kj} & \sqrt{(1-\alpha) P_{\mathrm{r}_k}} h_{kj} \end{bmatrix} \begin{bmatrix} n_{\mathrm{r}_j} \\ n_{\mathrm{r}_k} \\ x_s \end{bmatrix}. \end{aligned} \tag{24}$$

Assuming perfect detection of x_1 and x_2, SU_j will then try to decode x_s. An outage is declared if $R_{kj}^{AF} < R_{\mathrm{r}_j}$ where

$$R_{kj}^{AF} = \frac{1}{2}\log\left(1 + \frac{(1-\alpha) P_{\mathrm{r}_k} |h_{kj}|^2}{(\alpha P_{\mathrm{r}_k} |h_{kj}|^2 + 1)\sigma^2}\right). \tag{25}$$

The outage probability of SU_j could be then written as

$$\begin{aligned} \mathrm{P}_{\text{out}}^{\mathrm{SU}_j} &= \mathrm{P}\left[R_{kj}^{AF} < R_{\mathrm{r}_j}\right] \\ &= \mathrm{P}\left[\left(1 - \alpha\left(1 + \left(2^{2R_{\mathrm{r}_j}} - 1\right)\sigma^2\right)\right)\gamma_{kj} \geq \gamma_{\mathrm{r}_j}\right] \end{aligned} \tag{26}$$

where γ_{r_j} is defined in (17).

By taking $\alpha < \dfrac{1}{1 + \left(2^{2R_{\mathrm{r}_j}} - 1\right)\sigma^2}$, we can then write

$$\mathrm{P}_{\text{out}}^{\mathrm{SU}_j} = 1 - \exp\left(-\frac{\lambda_{kj}\gamma_{kj}}{1 - \alpha\left(1 + \left(2^{2R_{\mathrm{r}_j}} - 1\right)\sigma^2\right)}\right), \tag{27}$$

otherwise $\left(\text{if } \alpha \geq \dfrac{1}{1 + \left(2^{2R_{\mathrm{r}_j}} - 1\right)\sigma^2}\right)$, $\mathrm{P}_{\text{out}}^{\mathrm{SU}_j} = 1$.

By adopting the following notation

$$A_i = 2\sqrt{\frac{\sigma^2(2^{2R_i}-1)}{P_l P_{\mathrm{r}_k}\lambda_{1k}\lambda_{2k}}} \quad \text{and} \quad B_i = \frac{2^{2R_i}-1}{P_l \lambda_{lk}}, \tag{28}$$

the optimization problem in (2) could be then written for the AF relaying case such as

$$\begin{aligned} &\underset{\alpha}{\text{Minimize}} \quad 1 - \exp\left(-\frac{\lambda_{kj}\gamma_{kj}}{1 - \alpha\left(1 + \left(2^{2R_{\mathrm{r}_j}} - 1\right)\sigma^2\right)}\right) \\ &\text{subject to} \begin{cases} 1 - \dfrac{A_i}{\sqrt{\alpha}} \exp\left(-\dfrac{B_i}{\alpha\left(1-\alpha(1-\sigma^2)\right)^{-1}}\right) \\ \qquad \times K_1\left(\dfrac{A_i}{\sqrt{\alpha}}\right) \leq \mathrm{P}_{\text{out}_i} \\ 0 < \alpha < \min\limits_i \left(\dfrac{1}{1 + \left(2^{2R_{\mathrm{r}_j}} - 1\right)\sigma^2}\right) \end{cases} \end{aligned} \tag{29}$$

with $i \in \{1,2\}$.

To solve the optimization problem in (29), we write the Karush–Kuhn–Tucker (KKT) conditions and use Newton's method to find an optimal solution numerically.

C. Adaptive Relaying

For the adaptive relaying case, SU_k will try to decode the primary users symbols received in the first phase. If he succeeded in decoding both symbols, then he will broadcast a bitwise XOR of the two symbols as in the DF relaying case. But if the decoding was not successful, SU_k will amplify the received signal, add its own symbol and then broadcast it as in the AF relaying case.

An overall outage will occur at PU_i if $R_{ki}^{DF} < R_i$ in case SU_k did decode x_1 and x_2 or if $R_{ki}^{AF} < R_i$ in case he did not. The outage probability of PU_i could be then written for the adaptive relaying case such as

$$\begin{aligned} \mathrm{P}_{\text{out}}^{\mathrm{PU}_i} &= \mathrm{P}[R_{1k} \geq R_1]\,\mathrm{P}[R_{2k} \geq R_2]\,\mathrm{P}\left[R_{ki}^{DF} < R_i\right] \\ &\quad \times \mathrm{P}\left[R_{\sum} \geq R_1 + R_2 | R_{1k} \geq R_1, R_{2k} \geq R_2\right] \\ &+ \left\{1 - \mathrm{P}[R_{1k} \geq R_1]\,\mathrm{P}[R_{2k} \geq R_2]\,\mathrm{P}\left[R_{ki}^{AF} < R_i\right]\right\} \\ &\quad \times \mathrm{P}\left[R_{\sum} \geq R_1 + R_2 | R_{1k} \geq R_1, R_{2k} \geq R_2\right]. \end{aligned} \tag{30}$$

Similarly, the outage probability of SU_j is given by

$$\begin{aligned} \mathrm{P}_{\text{out}}^{\mathrm{SU}_j} &= \mathrm{P}[R_{1k} \geq R_1]\,\mathrm{P}[R_{2k} \geq R_2]\,\mathrm{P}\left[R_{kj}^{DF} < R_{\mathrm{r}_j}\right] \\ &\quad \times \mathrm{P}\left[R_{\sum} \geq R_1 + R_2 | R_{1k} \geq R_1, R_{2k} \geq R_2\right] \\ &+ \left\{1 - \mathrm{P}[R_{1k} \geq R_1]\,\mathrm{P}[R_{2k} \geq R_2]\,\mathrm{P}\left[R_{kj}^{AF} < R_{\mathrm{r}_j}\right]\right\} \\ &\quad \times \mathrm{P}\left[R_{\sum} \geq R_1 + R_2 | R_{1k} \geq R_1, R_{2k} \geq R_2\right]. \end{aligned} \tag{31}$$

Substituting (8), (10), (11), (17), (23) and (27) in (30) and (31), the optimization problem in (2) could be then written for the Adaptive relaying case such as

$$\underset{\alpha}{\text{Minimize}} \quad \left\{1-\exp\left(-\frac{\lambda_{kj}\gamma_{kj}}{1-\alpha\left(1+\left(2^{2R_{r_j}}-1\right)\sigma^2\right)}\right)\right\} \times(1-C)+C\left\{1-\exp\left(-\frac{\lambda_{kj}\gamma_{r_j}}{1-\alpha}\right)\right\}$$

subject to

$$\begin{cases} \left\{1-\frac{A_i}{\sqrt{\alpha}}\exp\left(-\frac{B_i}{\alpha\left(1-\alpha\left(1-\sigma^2\right)\right)^{-1}}\right)K_1\left(\frac{A_i}{\sqrt{\alpha}}\right)\right\} \\ \quad \times(1-C)+C\left\{1-\exp\left(-\frac{\lambda_{kj}\gamma_{r_j}}{1-\alpha}\right)\right\} \leq \mathrm{P}_{\mathrm{out}_i} \\ \max_i\left(1-2^{-2R_i}\right)<\alpha<\min_i\left(\frac{1}{1+\left(2^{2R_{r_j}}-1\right)\sigma^2}\right), \end{cases} \tag{32}$$

where C, A_i and B_i are defined in (12) and (28), and $i \in \{1,2\}$.

Solving the optimization problem in (32) requires the utilization of a numerical method. We use Newton's method in order to get the optimal solution.

IV. Numerical Examples

In this section, we provide some simulation results to show the performance of the proposed scheme under DF, AF and adaptive relaying using the optimal power allocation. We assume that the two primary users are located at distances $d_{1k}=d_{2k}=1$ from SU_k and $d_{1j}=d_{2j}=1.6$ from SU_j. Also, SU_j is placed at a distance of $d_{kj}=0.5$ from SU_k. The other system parameters are taken as follow, $\sigma^2=1$, $\beta=3$, $\lambda_{1k}=\lambda_{1j}=6$, $\lambda_{1j}=\lambda_{2j}=5$, $\lambda_{kj}=8$, $R_1=R_2=R_{r_j}=0.5$ and $P_1=P_2=10$ dB. Finally, the outage thresholds of the primary users are $\mathrm{P}_{\mathrm{out}_1}=\mathrm{P}_{\mathrm{out}_2}=0.07$.

Fig. 2 illustrates the outage probabilities of the two primary users and the secondary user SU_j as a function of the value of α. We assume that the simulated system uses the adaptive scheme and that the transmit power of SU_k is $P_{r_k}=20$ dB. As is evident, increasing the value of α improves the outage probabilities of the primary users while the performance of SU_k decreases. It can also be noticed that below the optimal value of α all the outage probabilities do change with very small values.

The optimal value of α versus the transmit power available at SU_k is shown in Fig. 3. For small values of P_{r_k}, the secondary transmitter has to allocate a maximum fraction of its power to satisfy the outage probabilities of the primary users. For large values of P_{r_k}, the optimal value of α becomes constant for the three relaying schemes. It should be noted that, for this figure, we attribute the value 1 to α_{opt} when no optimal solution exists.

We plot in Fig. 4 the outage probability of PU_1 as a function of P_{r_k}. For relatively small values of P_{r_k}, the primary outage probability obtained when applying AF relaying is better than the one obtained through DF relaying. However, when we increase the value of P_{r_k}, DF relaying outperforms clearly AF relaying. Obviously, the proposed adaptive relaying outperforms the other two schemes.

Finally, the outage probability of the secondary user SU_j versus the primary transmit power P_1 is illustrated in Fig. 5.

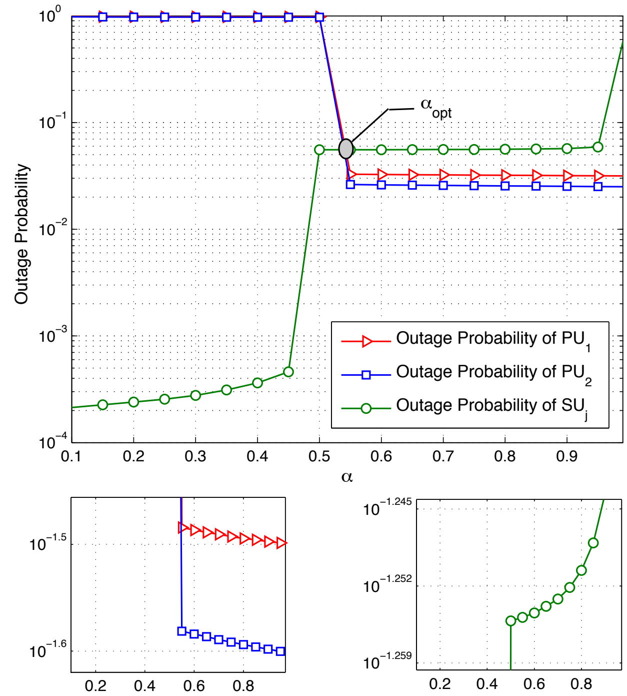

Fig. 2. The outage probability of the primary and secondary users in function of the fraction of power used by SU_k to rely the PUs .

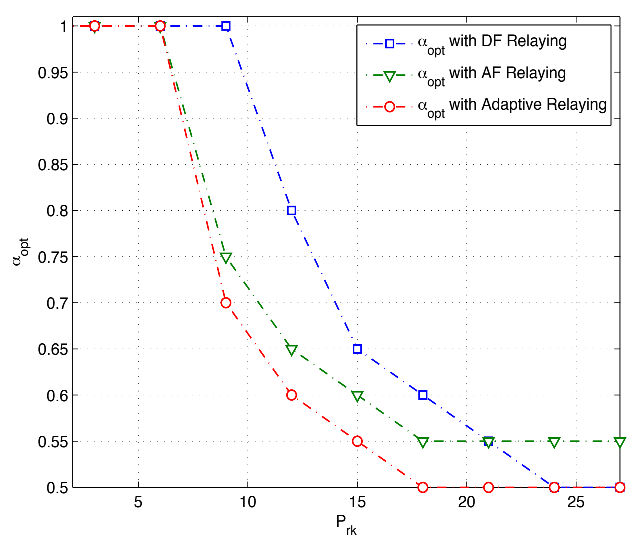

Fig. 3. The optimal value of α for different values of the available power at SU_k for the three different analyzed relaying techniques.

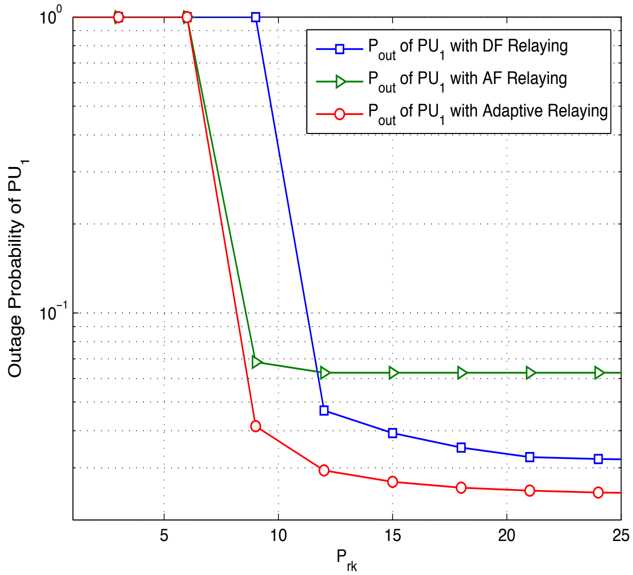

Fig. 4. The outage probability of PU_1 as a function of the available power at SU_k for the three different analyzed relaying techniques.

Here, the available transmit power at SU_k is $P_{r_k} = 10$ dB and we take $P_1 = P_2$. It can be observed that the DF relaying scheme outperforms clearly the AF one especially for high values of P_1 since the former scheme decodes primary signals with higher probability. Obviously, adaptive relaying outperforms the other two schemes and it is therefore the best relaying option. Also, the three schemes take benefit from the increase of the strength of the primary signals in order to improve the performances of the secondary link.

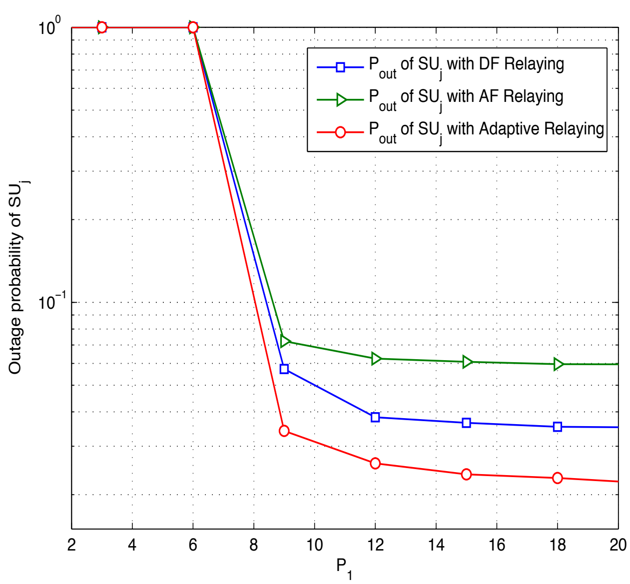

Fig. 5. The outage probability of SU_j as a function of the available power at the primary system for the three different analyzed relaying techniques.

V. Conclusion

In this work, we considered an overlay cognitive radio network comprising two primary users that communicate via a two-phase two-way relaying secondary user. Our approach was based on the minimization of the outage probability of the secondary user given outage constraints on the primary system. Closed-form expressions for the outage probabilities of the primary and secondary users were derived for decode-and-forward, amplify-and-forward and adaptive relaying. Besides, an optimal power allocation scheme at the secondary relay has been analyzed. As an extension to this work, it would be of interest to consider the communication scenario over different fading channels when multiple secondary users are present in the primary users' area.

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