# Improving the Throughput of Cognitive Radio Networks Using the Broadcast Approach

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***Abstract*—We study the impact of adopting a multi layer coding (MLC) strategy, i.e., the so-called broadcast approach (BA) on the throughput of Cognitive Radio (CR) spectrum sharing systems for general fading channels. First, we consider a scenario where the secondary transmitter, a part from the statistics, has no channel state information (CSI) of the cross link and its own link. We show that using BA improves the cognitive achievable rate compared to the outage rate provided by a single layer coding (SLC). In addition, we, also, observe numerically that 2-Layer coding achieves most of the gain. Then, we consider a situation where the secondary transmitter has a partial CSI about its own link through quantized CSI. Again, we compute the secondary achievable rate adopting the BA and highlight the improvement over SLC. Numerical results show that the advantage of MLC decreases as the rate of the feedback link increases.**

***Index Terms*—Underlay cognitive radio, achievable rate, broadcast approach, quantized CSI, fading channels.**

## I. Introduction

In wireless communication, when a delay constraint is to be respected, performance limits of a wireless communication cannot be captured by measuring the ergodic capacity anymore since the code symbols cannot be arbitrary long and thus the ergodic properties of the channel are not revealed. This situation is generally modeled by letting the fading coefficient constant over a long transmission block over which communication should take place. In this case, the notion of capacity-versus-outage is more appropriate and without perfect CSI at the transmitter, the capacity in the Shannon sense is equal to zero since no matter how small the rate we are communicating at, there is no guarantee that this rate can be conveyed reliably to the receiver [1]–[3].

The capacity-versus-outage performance is defined as the probability that the channel cannot support a given rate, i.e., the outage capacity. If on the contrary, perfect CSI is available at the transmitter, a positive rate at zero outage may be achieved through power control. Furthermore, having CSI, the transmitter may attempt to invert the channel, if its power budget supports such a policy, thus eliminating the fading which opens the door to the notion of delay-limited capacity. The later capacity is formally defined as the rate that corresponds to zero outage [4]. Delay-limited capacity can also be understood as the capacity of a compound channel where the CSI associated with the fading is the parameter governing the transition probability of that channel. Weather ergodic, outage or delay-limited, the capacity expression depends on the CSI at the receiver and at the transmitter. For most cases, CSI at the transmitter gives an advantage only through power control. While this advantage is not significant under a short term power constraint, it is remarkably considerable under long term power constraint, see e.g., [5]–[9] and references therein. Therefore, we focus, next, on communication scenarios where the receiver(s) is perfectly aware of the channel gain CSI, whereas, the transmitter is not. However, we assume that the latter is either only aware of the statistics of the fading channel or is provided with a quantized version of the actual channel gain through an error-free low-rate feedback link. As the feedback link is of low rate, one can reasonably justify the error-free assumption of the feedback. Furthermore, we assume that communication should take place in a limited number of coherence block, say $M$, due to a stringent delay constraint. For a sake of simplification, we will assume that $M = 1$, i.e., coding over one coherence block.

The broadcast approach (BA) for a compound channel has been first introduced by Cover [10]. This approach facilitates to deliver information rates which depend on the actual realization of the channel and without the transmitter being aware of which state the channel is. The BA has been pursued in the case of flat-fading Gaussian channel with no fading dynamics, i.e., the quasi-static channel model, in [11], [12]. We present the interpretation of single-user broadcasting as a multi-layer coding (MLC) e.g., [13], [14]. We introduce the BA in the cognitive context by considering a spectrum sharing setting that incorporates partial side information, among others. We argue that this scenario has not yet been given full consideration in existing state of the art.

The rest of the paper is organized as follows. In Section II, the adopted system model is presented. In section III the

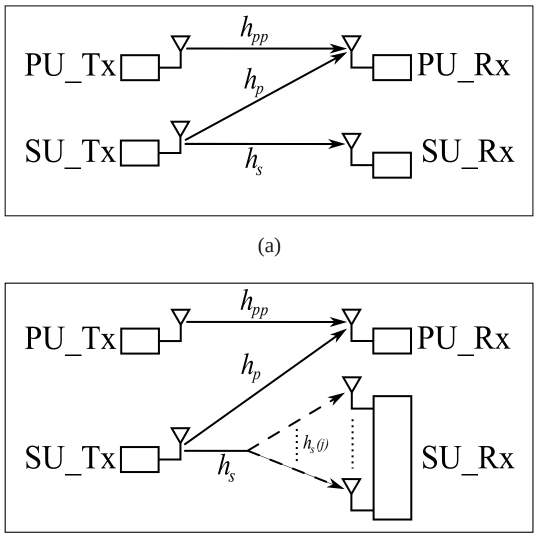


Figure 1. (a) A spectrum sharing channel model (b) Equivalent model adopting the broadcast approach.

impact of the BA on CR setting is studied whereas in section IV the quantized CSI policy is introduced. Numerical results and their interpretations are presented in Section V. Finally, the paper is concluded in Section VI.

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## II. System Model

We consider a spectrum sharing communication scenario as depicted in Fig.1(a), where a secondary user transmitter (SU_Tx) is communicating with a secondary user receiver (SU_Rx), under certain constraints that will be defined later, through a licensed bandwidth occupied by a primary user (PU). The signal received at the SU is given by:

$$y_s(i) = h_s\, x(i) + n_s(i), \tag{1}$$

where $i$ is the discrete-time index, i=1,...,T, where T is the coherence time assumed here equal to the symbol block length, for simplicity; the symbol $x(i)$ represents the channel input, $h_s$ is the complex channel gain, and $n_s(i)$ is a circularly symmetric complex Gaussian noise with zero mean and unit variance, i.e., $\mathcal{CN}(0,1)$ and is independent of $h_s$. The channel gains in Fig.1, $h_s$, $h_p$ and $h_{pp}$ are assumed to have continuous probability distribution function (PDF)'s denoted $f_{\mathbf{h}_s}(h_s)$, $f_{\mathbf{h}_p}(h_p)$ and $f_{\mathbf{h}_{pp}}(h_{pp})$, respectively. We consider a short term power constraint, $P_{ST}$ (excluding power adaptation over different transmission blocks). The channel model (1) is also known as the quasi-static flat fading channel model suitable for delay-constrained communications. The SU_Tx is ignorant about the instantaneous CSI of the SU channel gain, $h_s$ and the cross link $h_p$. However, it is provided their statistics through $f_{\mathbf{h}_s}(h_s)$ and $f_{\mathbf{h}_p}(h_p)$. This setting describes a spectrum sharing communication where the SU_Rx is able to estimate its own channel and in the absence of a potential incentive and/or a cooperation protocol, the PU is not willing to feed back the channel gain $h_p$ to the SU_Tx. It is clear that in the absence of the cross link CSI, the SU_Tx is unable to protect fully the primary receiver (PU_Rx) against interference. Instead, the SU relies on the statistics of $h_p$ to establish a reliable communication without prejudicing the PU [15]. That is, the probability that the interference power at the PU be above a given positive threshold should be kept as small as possible, i.e.,

$$\mathrm{Prob}\Big\{P\,|h_p|^2 \geq Q_{peak}\Big\} \leq \varepsilon, \tag{2}$$

where $P$ is the transmit power at the SU_Tx, $Q_{peak}$ is the interference level tolerated by the PU_Rx and where $\varepsilon > 0$ is the threshold that may be regarded as the percentage of time that the SU is allowed to exceed $Q_{peak}$. Constraint (2) aims at reducing the instantaneous interference power at the PU_Rx for a given secondary link channel realization $h_p$. We could have set (2) more reliably as a constraint on the Signal to Interference plus Noise Ratio ($SINR$) at the PU_Rx, however this would have required the knowledge of both the power at the primary transmitter (PU_Tx) and the statistics of the primary link $h_{pp}$. Note that (2) is equivalent to:

$$P \leq P_{peak}, \tag{3}$$

where $P_{peak} = \frac{Q_{peak}}{F^{-1}_{|h_p|^2}(1-\varepsilon)}$ and $F^{-1}_{|h_p|^2}(\cdot)$ is the inverse cumulative distribution function (CDF) of $|h_p|^2$. Constraint (3) can be interpreted as a peak transmit-power constraint dictated by the interference constraint of the PU_Rx.

Without cognitive constraint, the classical coding strategy in this case consists of a single-codebook, on-off power transmission scheme, the threshold of which is chosen such that the average throughput over all coherence blocks is maximized. This is known as the outage approach (OA) (also called outage capacity) or SLC. Comes the cognitive constraint (either statistical or on the average), several authors have adopted such an approach with proper adaptation to CR settings [3], [16], [17]. To the best of our knowledge, performance of MLC in CR settings has not been evaluated previously which constitutes the main motivation of this paper.

## III. The Broadcast Approach in CR Settings

### A. Preliminary

An equivalent broadcast channel of the secondary link is illustrated in Fig.1(b). During each channel realization $h_s$, the transmitter sends an infinite number of layers of coded information. The receiver is regarded as continuum of ordered users, each decoding a code layer if the channel realization allows it. The ordering here is guaranteed by the nature of the Gaussian broadcast channel. Each of the users has to decode a fractional rate, $dR$, which is not equal for different users and rather depends on the user index. The first user (the weakest of all users or equivalently the one supporting the lowest data rate) decodes only its own $dR$. We are interested in maximizing the average achievable rate of the SU, in nats per channel use (npcu), over many transmission blocks. The broadcast approach for a single user (SU) channel which relies on assuming that there is an infinite number of ordered receivers. Let us consider the following flat fading channel model, at time instant $i$, $i = 1, \ldots, \infty$: $y_i = h_s\, x_i + n_i$, where $y_i$ is the received symbols, $h_s$ is the complex channel gain, $x_i$ is the complex transmitted symbol and $n_i$ is $\mathcal{CN}(0, 1)$. Each realization of $h_s$, in (1), is associated to an achievable rate and we are interested in the average achievable rate for various independent transmission blocks. Obviously, if perfect CSI is available at the transmitter, the expected rate over many transmission blocks and under a short term power constraint $\underset{x}{\mathbb{E}}\left[|x|^2\right] \leq P_{ST}$, is given by: $C_{erg} = \underset{\gamma_s}{\mathbb{E}}[\log(1 + P_{ST}\, \gamma_s)]$, where $\gamma_s = |h_s|^2$. Since the actual channel realization is $h_s^{(j)}$, then any rate slightly higher than $\log\left(1 + P_{ST}\, \gamma_s^{(j)}\right)$ cannot be decoded reliably and thus users $j+1$, $j+2$,... achieve nothing. Hence, the total achievable rate for channel realization $h^{(j)}$ is the integral of $dR$ over all users up to $j$. For a continuous fading power, the total achievable rate for a fading realization $\gamma_s$ is the integration of all the fractional achievable rates of all users with successful layer decoding: $R(\gamma_s) = \int_0^{\gamma_s} \frac{u\,\rho(u)}{1+u\,I(u)}\, \mathrm{d}u$, where $u\,\rho(u)\,\mathrm{d}u$ is the receive power of a layer parameterized by $u$ and intended to the user indexed by the channel gain power $u$; and $I(u)$ is the interference induced by information data intended to users indexed by channel gain power $v > u$. The later data streams cannot be decoded reliably and thus constitute an additional interference noise

$$I(\gamma_s) = \int_{\gamma_s}^{\infty} \rho(u)\, \mathrm{d}u. \tag{4}$$

In particular, the total available power at the transmitter is given by: $P_{ST} = \int_0^{\infty} \rho(u)\, \mathrm{d}u$.Therefore, the maximum average rate achieved by this broadcast approach over many transmission blocks, $R_{ba}$, is given by $R_{ba} = \int_0^{\infty} R(u)\, f_s(u)\, \mathrm{d}u$, where $f_s(u)$ is the PDF of the fading power. Maximizing the average rate over all transmit power strategies is formulated as

$$R_{ba}^{max} = \max_{I(u)} \int_0^{\infty} R(u)\, f_s(u)\, \mathrm{d}u. \tag{5}$$

This optimization problem has been solved in [12], where it is found that the optimal $I(u)$ is given by:

$$I(u) = \begin{cases} \frac{1 - F_s(u) - u\, f_s(u)}{u^2\, f_s(u)} & \text{if } u_0 \leq u \leq u_1 \\ 0 & \text{else,} \end{cases} \tag{6}$$

where $F_s(u)$ is CDF of the fading power; and where $u_0$ is determined by $I(u_0) = P$ and $u_1$ by $I(u_1) = 0$. From (6), the optimal transmit power distribution $\rho(\gamma_s)$ can be derived using (4).

### B. The Cognitive Radio Setting

In regard of the cognitive constraint (3), it turns out that in this simple setting, the problem at hand is equivalent to maximizing the average achievable rate of the secondary link without cognitive constraint, but under a modified power constraint $P \leq \min\left(P_{ST}, P_{peak}\right) \triangleq P_0$. Therefore, the achievable rate using the BA holds with $P$ substituted by $P_0$, $f_s(\cdot)$ by $f_{\mathbf{h}_s}(\cdot)$ and $F_s(\cdot)$ by $F_{\mathbf{h}_s}(\cdot)$. Naturally, even in CR settings, the BA outperforms the standard OA in terms of average achievable rate. Recall that the OA consists of transmitting at a fixed rate and whenever the channel power is above a certain threshold, successful decoding is possible, otherwise, an outage is declared. Clearly, the OA's average rate corresponds to the optimization (5) over a restrained subspace of transmit power policies defined as $\rho(\gamma_s) = P\, \delta(\gamma_s - \gamma_{th})$, where $\gamma_{th}$ is a threshold defining the outage event and where $\delta(\cdot)$ is the Dirac delta function.

### C. 2-Layer Coding

Although the BA is formally described with infinite code-layers for fading with continuous power gains, the performance of few code-layers (e.g 2 layers) is very close to the optimal performance [18], [19]. To this end, we study the case of 2-layer coding. The SU_Tx sends a superposition of two codewords defined by two rates; $R_1$ and $R_2$ corresponding to the channels $\gamma_1$ and $\gamma_2$ ($\gamma_1 \leq \gamma_2$), respectively. In this design, $\gamma_1$ and $\gamma_2$ are the reconstruction channels above which the signal is no more decodable. Each rate is rewarded by the probability that the actual channel gain is higher than $\gamma_i$, $i = 1, 2$. and the total rate is, hence, written as

$$R_{s,L=2} = [1 - F_{\mathbf{h}_s}(\gamma_1)] \log\left(1 + \frac{\gamma_1 P_1}{1 + \gamma_1 P_2}\right) + [1 - F_{\mathbf{h}_s}(\gamma_2)] \log(1 + \gamma_2 P_2). \tag{7}$$

where $P_1$ and $P_2$ are the powers allocated to each layer. By adopting a short term power and statistical interference constraints, these powers are subject to the following inequalities

$$P_1 + P_2 \leq P_{ST}, \tag{8}$$

$$\mathrm{Prob}\left\{\gamma_s(P_1 + P_2) \geq Q_{peak}\right\} \leq \varepsilon. \tag{9}$$

Recall that (8) and (9) can be reduced to:

$$P_1 + P_2 \leq P_0, \tag{10}$$

In order to find the problem solution, i.e optimal gains and powers that maximizes the rate (7), we define the following optimization problem:

$$\max_{\gamma_1,\gamma_2,P_1,P_2} R_{s,L=2}, \tag{11}$$
$$\text{s.t} \quad (10). \tag{12}$$

Similar optimization problems have been studied in the literature, i.e, [19], [20] where the optimal power is determined with a method based on the water-filling power allocation as follows:

$$P_i = \{x \text{ such as } J(x) = i\}, \text{ for } i = 1, 2, \tag{13}$$

where $J(x)$ is an index function defined as $J(x) = arg\max_i \frac{1-F_{\mathbf{h}_s}(\gamma_i)}{\frac{1}{\gamma_i}-x} - \lambda$,and $\lambda$ is the lagrangian multiplier associated to the constraint (12), $\lambda$ is a constant $\forall i$ and is computed such as $\lambda = \max_i \frac{1-F_{\mathbf{h}_s}(\gamma_i)}{\frac{1}{\gamma_i}-P_0}$.Meanwhile, in order to find the optimal reconstruction points, we need to compute the Lagrangian function of the optimization problem (11) which is defined as $\mathcal{L} = R_{s,L=2} - \lambda(P_1 + P_2 - P_0)$.Then, we compute its partial derivatives with respect to $\gamma_1$ and $\gamma_2$ and determine their roots, respectively. Since the solution cannot be found immediately due to the dependency between all the parameters, we proceed by a heuristic algorithm where we fix a random starting point. Then, after several iterations we verify the convergence using, for example, the uniform/infinity norm (see **Algorithm 1**). Note that depending on the fading distribution, $\frac{\partial\mathcal{L}}{\partial\gamma_i}$ is easily expressed and finding $\gamma_1^{(i+1)}$ is done using a simple root-finding algorithm. Where $\epsilon_0 > 0$ determines the stopping criteria of the algorithm. Note that, for given $\gamma_1$ and $\gamma_2$, the above optimization problem is equivalent to assigning powers to users in a fading broadcast channel where the probability of success of each layer is the reward assigned to each user [20], [21].

**Algorithm 1** 2-Layer Coding with no CSI

Initialize by starting points: $\gamma_1^{(0)}, \gamma_2^{(0)}, P_1^{(0)}, P_2^{(0)}$;
$i = 0$
**repeat**
  compute $\gamma_1^{(i+1)}$ such as $\frac{\partial\mathcal{L}}{\partial\gamma_1}|_{\gamma_1=\gamma_1^{(i+1)}} = 0$;
  compute $\gamma_2^{(i+1)}$ such as $\frac{\partial\mathcal{L}}{\partial\gamma_2}|_{\gamma_2=\gamma_2^{(i+1)}} = 0$;
  compute $P_1^{(i+1)}$ and $P_2^{(i+1)}$ according to (13) with the updated $\gamma_1^{(i+1)}$ and $\gamma_2^{(i+1)}$;
  $i \leftarrow i + 1$
**until** convergence ($|R_{s,L=2}^{(i+1)} - R_{s,L=2}^{(i)}| < \epsilon_0$).
$R_{s,L=2}^* = R_{s,L=2}(\gamma_1^{(i)}, \gamma_2^{(i)}, P_1^{(i)}, P_2^{(i)})$.

So far, we have assumed that the transmitter is not aware of the actual channel gain. Clearly, this represents a worst case scenario that provides a lower bound on the average secondary throughput. However, in most wireless systems, there exists a feedback link from the receiver to the transmitter that may be used to provide a quantized version of the actual CSI to the transmitter. Below, we study the impact of such a feedback on the performance of MLC in CR networks.

## IV. The Broadcast Approach under Quantized CSI

Assume now that the channel gain $\gamma_s$ is subdivided into K regions $[\tau_i, \tau_{i+1}[$ , $i = 0, \cdots, K$, where $\tau_0 = 0$ and $\tau_K = \infty$, for convenience. We first introduce the SLC with quantized CSI under cognitive constraint as a benchmark.

### *A. Single-Layer Coding*

For a region of index $i$( $i = 0, \cdots, K$), the SU_Tx use the power $P_0$ to transmit the signal with a rate $R_i = \log(1+\gamma_i P_0)$, where $\gamma_i$ is the reconstruction point of region $i$. This strategy provides SU a reward equal to the probability of success, i.e., the probability that $\gamma_i \in [\tau_i, \tau_{i+1}[$. Consequently, by considering all the regions, the total expected rate of the SU, with quantized CSI, is determined by solving the following optimization problem

$$R_{s,K} = \max_{\{\tau_i,\gamma_i\}} \sum_{i=1}^{K} [F_{\mathbf{h}_s}(\tau_{i+1}) - F_{\mathbf{h}_s}(\gamma_i)] \log(1 + \gamma_i P_0). \tag{14}$$
$$\text{s.t } \tau_{i+1} \geq \gamma_i \geq \tau_i \text{ for } i = 1, \cdots, K. \tag{15}$$

The optimal $\tau_i$ is the furthest from $\gamma_{i-1}$ as $F_{\mathbf{h}_s}(\cdot)$ is an increasing function. Hence, the optimal $\tau_i$ is such that

$$\tau_i = \gamma_i \quad \text{for } i = 1, \cdots, K. \tag{16}$$

Adopting (16) leads to a simpler problem with reduced number of variable. We then compute the Lagrangian of the problem which gives the following optimality condition, for $i = 1, \cdots, K$;

$$\gamma_{i+1} = F_{\mathbf{h}_s}^{-1}\left(F_{\mathbf{h}_s}(\gamma_i) + f_{\mathbf{h}_s}(\gamma_i)\frac{1+\gamma_i P_0}{P_0}\log\left(\frac{1+\gamma_i P_0}{1+\gamma_{i-1}P_0}\right)\right), \tag{17}$$

where $F_{\mathbf{h}_s}^{-1}(\cdot)$ is the inverse CDF of $|h_s|^2$. The conditions in (17) are simple to be solved since $\gamma_i$ can be expressed by $\gamma_1$, for $i = 2, \cdots, K$.

### *B. Multi-Layer Coding*

The fading channels with perfect CSI at the transmitter and at the receiver is generally well modeled by parallel Gaussian channels, where each of these channels represents a fading state [22]. With this approach, the degradedness of the additive white Gaussian noise (AWGN) broadcast channel is exploited in order to increase the expected cognitive rate. Although in our setting, we do not assume perfect CSI at the transmitter, SU_Tx looks at the reconstruction point $\gamma_i$ as if it is the actual channel gain in the region $i$, $i = 1, \ldots, K$. Meanwhile, the SU_Rx performs a successive decoding scheme based on the rate of the actual channel gain. In our scheme, in addition to the $K$ regions of channel gain, the SU_Tx transmits a superposition of $L$ codewords in each region. For a region of index $i$, $i = 1, \cdots, K$ and under a codeword $j$, $j = 1, \cdots, L$, the corresponding rate based on a reconstruction channel gain $\gamma_{ij}$ is given by $R_{ij} = \log\left(1 + \frac{\gamma_{ij} P_{ij}}{1+\gamma_{ij}\sum_{k=j+1}^{L} P_{ik}}\right)$.Where we assume that the set of $\gamma_{ij}$'s is ordered such as $\gamma_{i1} \leq \cdots \leq \gamma_{iL}$ $\forall i = 1, \cdots, K$. Recall that for any $\gamma_s$ such as $\gamma_s \geq \gamma_{ij}$, the signal is decoded. Hence, the total secondary rate with MLC and quantized CSI is given by

$$R_{s,K,L} = \max_{\{\tau_i, \gamma_{ij}, P_{ij}\}} \sum_{i=1}^{K} \sum_{j=1}^{L} \left[F_{\mathbf{h}_s}(\tau_{i+1}) - F_{\mathbf{h}_s}(\gamma_{ij})\right] \times \log\left(1 + \frac{\gamma_{ij} P_{ij}}{1 + \gamma_{ij} \sum_{k=j+1}^{L} P_{ik}}\right), \quad (18)$$

$$\text{s.t } \tau_{i+1} \geq \gamma_{ij}, \qquad \gamma_{i1} \geq \tau_i, \qquad \gamma_{i(j+1)} \geq \gamma_{ij} \quad (19)$$

$$\sum_{j=1}^{L} P_{ij} \leq P_0 = \min\left(P_{ST}, P_{peak}\right). \quad (20)$$

Note that, for the same reasons that have justified (16), the optimal set of $\tau_i$ can be picked such as $\tau_i = \gamma_i$ for $i = 1, \cdots, K$. Similarly to the no CSI problem in Section IV-A a set of optimality conditions are obtained as follows; for $i = 1, \cdots, K$;

$$F_{\mathbf{h}_s}(\gamma_{(i+1)1}) = F_{\mathbf{h}_s}(\gamma_{i1}) + f_{\mathbf{h}_s}(\gamma_{i1}) \frac{(1 + \gamma_{i1} P_0)(1 + \gamma_{i1}(P_0 - P_{i1}))}{P_{i1}} \left(R_{i1} - \sum_{j=1}^{L} R_{(i-1)j}\right). \quad (21)$$

A similar problem was studied in [19] and the solution was computed using an iterative algorithm (See **Algorithm 2**) consisting of; first, choosing a random starting points, respecting the constraints, then fixing the $\gamma_{i1}$'s and the $P_{ij}$'s, and compute the optimal $\gamma_{ij}$ and $1 < j \leq L$. The next step is fixing the $\gamma_{ij}$'s.computing the optimal $P_{ij}$'s according to (13). Finally, the optimality condition (21) is used to compute the optimal $\gamma_{i1}$'s when the $\gamma_{ij}$ and the $P_{ij}$'s are fixed. Then the loop is repeated until convergence of the rate $R_{s,K,L}$.

**Algorithm 2** Multi-layer coding with quantized CSI

Initialize by starting points: $\gamma_{ij}^{(0)}, P_{ij}^{(0)}$;
$k = 0$
**repeat**
  fix $\gamma_{i1}^{(k)}, P_{ij}^{(k)}$ and compute $\gamma_{ij}^{(k)}, 1 < j$ ;
  fix $\gamma_{ij}^{(k)}, \forall j$ and compute $P_{ij}^{(k+1)}$ using (13);
  fix $\gamma_{i1}^{(k)}, 1 < j, P_{ij}^{(k+1)}$ and compute $\gamma_{i1}^{(k+1)}$ ;
  $k \leftarrow k + 1$
**until** convergence ($|R_{s,K,L}^{(i+1)} - R_{s,K,L}^{(i)}| < \epsilon_0$).
$R_{s,K,L}^{*} = R_{s,K,L}(\gamma_{ij}^{(k)}, P_{ij}^{(k)})$.

## V. Numerical Results

We apply the previous results to a normalized Rayleigh fading channel, i.e., $f_{\mathbf{h}_s}(x) = f_{\mathbf{h}_p}(x) = e^{-x}$ and $F_{\mathbf{h}_s}(x) = F_{\mathbf{h}_p}(x) = 1 - e^{-x}$, the maximum average achievable secondary rate using the BA is given by [12]:

$$R_{s,ba}^{max} = 2Ei(\gamma_0) - 2Ei(1) - (e^{-\gamma_0} - e^{-1}), \quad (22)$$

where $\gamma_0 = \frac{2}{1+\sqrt{1+4P_0}}$ and $Ei(x) = \int_x^{\infty} \frac{e^{-t}}{t}\,dt$ is the exponential integral function.

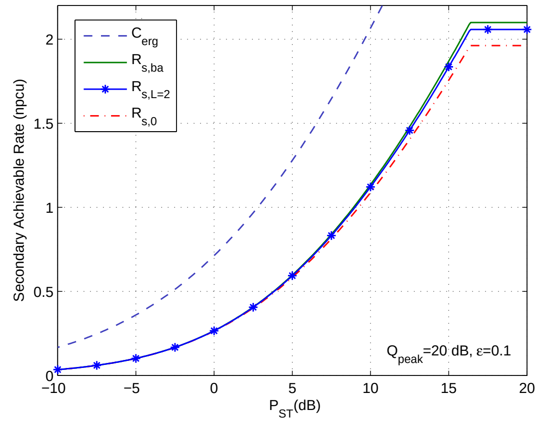


Figure 2. Secondary user average achievable rate using the broadcast approach, 2-layer coding and the standard outage approach.

An evaluation of the secondary achievable rate using the BA, $R_{s,ba}^{max}$ given by (22) versus $P_s$, is displayed in Fig. 2, for $Q_{peak} = 20$ dB and $\varepsilon = 0.1$. Also shown in Fig. 2 is the average achievable rate using the standard OA, $R_{s,o}^{max}$ given by: $R_{s,o}^{max} = e^{-\gamma_{th}} \log(1 + \gamma_{th} P_0)$,where $\gamma_{th}$ is the outage threshold and which is determined by solving the equation $(1 + \gamma_{th} P_0) \log(1 + \gamma_{th} P_0) = P_0$.It is clear from Fig. 2 that the BA outperforms the standard OA in terms of average achievable rate, especially at high SNR. This trends holds true for different values of $Q_{peak}$ and $\varepsilon$. Interestingly, even the less complex 2-layer coding (L=2) outperforms the classical OA, as shown in Fig. 2.

In Fig.3 the achievable rate is presented as a function of $P_{ST}$ for different number of feedback bits (i.e $K = 1, 2, 4, 10$) in order to display the performance of the OA with quantized secondary CSI. We note that with quantized CSI feedback the expected rate is enhanced but the gain in rate is becoming smaller as the number of region increases.

In Fig.4, the secondary rate as function of $P_{ST}$ with the quantized CSI and MLC is presented. We note that when K increases, the gain of MLC decreases. Hence, the availability of a quantized CSI at the transmitter undermines the performance gain of the BA. As shown in Fig.4, there is no need for a MLC when K exceeds 4, since both BA and OA perform quasi similarly in this case.

## VI. Conclusion

In this paper, we have studied the achievable rate of the secondary link in a cognitive radio (CR) spectrum sharing

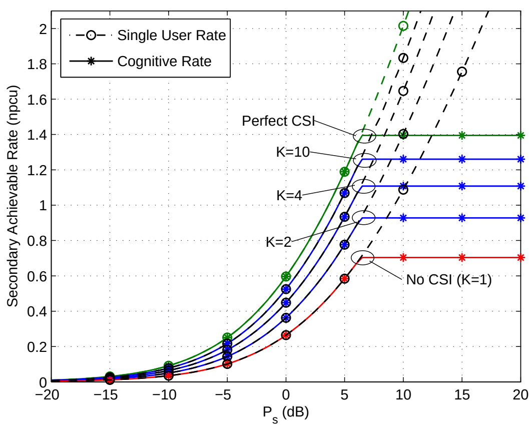


Figure 3. Achievable rate of the secondary user with quantized CSI versus $P_{ST}$, for different number of feedback bits.

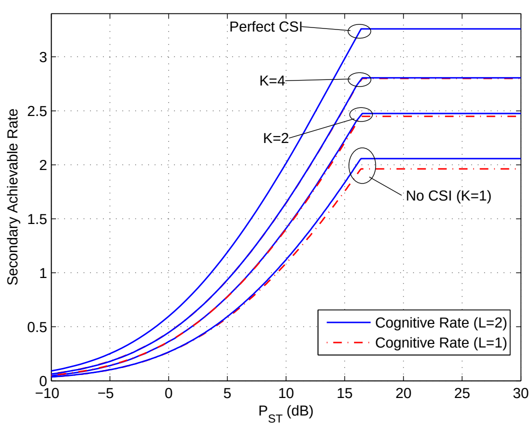


Figure 4. Average achievable rate by the secondary user using quantized CSI and multi-rate coding.

scenario when a broadcast approach (BA) is adopted. First, using an infinite-layer coding, we have highlighted the corresponding improvement on the expected secondary rate when no CSI is available at the transmitter. Then, we have shown numerically that a 2-layer coding achieves most of this gain, for Rayleigh fading channels. Next, we have considered a situation where the secondary user transmitter is aware of a quantized version of the actual CSI, and evaluated the average throughput in a Rayleigh fading context; we have shown that the secondary expected rate, with a single-layer coding, is enhanced considerably even with few bits of CSI feedback. Adopting a BA in this context further improves the throughput and the BA outperforms again the OA for a given number of feedback. However, as the number of feedback bits increases, the relative improvement of the BA becomes limited.

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