On the Outage Capacity of the M-Block Fading Channel at Low-Power Regime

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***Abstract*—Outage performance of the M-block fading with additive white Gaussian noise (BF-AWGN) is investigated at low-power regime. We consider delay-constrained constant-rate communications with perfect channel state information (CSI) at both the transmitter and the receiver (CSI-TR), under a short-term power constraint. We show that selection diversity that allocates all the power to the strongest block is asymptotically optimal. Then, we provide a simple characterization of the outage probability in the regime of interest. We quantify the reward due to CSI-TR over the constant-rate constant-power scheme and show that this reward increases with the delay constraint. For instance, for Rayleigh fading, we find that a power gain up to 4.3 dB is achievable.**

***Index Terms*—Outage capacity, low-power, block-fading channels, CSI-TR.**

I. Introduction

In many situations, communications should happen at low-power regime. This includes for instance, wideband communications where the power per sub-band can be very low, sensor network communications, free-space optical communications, etc. The low-power regime framework is also useful to study communication systems where the power per degrees of freedom (time, space or frequency) is low. A wide body of work exists by now focusing on communications at low-power regime either from performance limits and signaling design perspectives, e.g., [1]–[5], or from more practical perspectives related to deployment, implementation and challenges, e.g., [6], [7], to cite only few.

In delay-unconstrained communication, the ergodic capacity is the most popular performance measure. Characterization of performance limits in this case takes the form of deriving the capacity behavior at asymptotically low-power regime depending on the channel state information (CSI) at the receiver (CSI-R) or/and at the transmitter (CSI-T), e.g., [8]–[13]. On the contrary, for delay-constrained communications, much less papers have focused on performance in the low-power regime [14]. In [15] (see also [16, Chap. 5]), it has been shown that the role of fading on outage performance is much more important in the low-power regime than in the high-power regime thereby strongly motivating performance limits derivations of delay-constrained communications in the low-power regime.

In this paper, we consider delay-constrained communications over a block fading (BF) channel with additive white Gaussian noise (AWGN). We assume that communication should happen within a limited number of blocks M that constitute a frame. Despite the availability of perfect CSI at both the transmitter and the receiver (CSI-TR), a constant-rate coding scheme is imposed. However, power control over the M blocks is possible. This problem has been solved in [17]. Nevertheless, as we argue next (Section II), the solution does not lend itself to nice analytical formulations, thus preventing generally gaining insight on the impact of system parameters on outage performance. Our focus is on low-power regime which is formally defined next. We show that the minimum outage takes a simple form. More specifically, we show that for the setting described above, selection diversity is asymptotically optimal. Furthermore, we highlight the value of CSI-TR over CSI-R in terms of outage probability in function of the number of blocks M and for different types of fading. Perhaps the most striking result of this paper is the fact that even under a short-term power constraint, the gain in terms of outage probability due to CSI-TR over CSI-R increases unboundedly with M, provided that the targeted outage performance is low.[1]

II. System Model AND Related Background

We consider a block-fading with additive white Gaussian noise (BF-AWGN) channel comprised of M blocks. Each block contains N symbols that undergo the same fading state which is constant, but random. A codeword of length $M_N = MN$ spans M blocks. The n-th, $n = 1, \ldots, N$, channel output at coherence block m, $m = 1, \ldots, M$, is expressed by:

$$y(m,n) = h_m \; x(m,n) + z(m,n), \tag{1}$$

where $x(m,n) \in \mathbb{R}$ is the n-th transmitted symbol at time coherence m; $h_m \in \mathbb{R}$ is a zero-mean and unit-variance channel gain during block m and $z(m,n) \in \mathbb{R}$ is the AWGN with zero-mean and unite-variance. The fading process $\{h_m\}$ is assumed to be independent (not necessarily identically distributed) across the M blocks. We assume perfect CSI-TR. For convenience, we let $\gamma_m = |h_m|^2$, $f_m(\cdot)$ its probability density function (PDF), and $F_m(\cdot)$ its cumulative distribution functions (CDF), $m = 1, \ldots, M$. We assume that $F_m(\cdot)$ is defined

[1] *Notations:* The logarithms $\log(x)$ is the natural logarithm of x. We say that $f(x) \overset{a}{\approx} g(x)$ if and only if $\lim_{x\to a} \frac{f(x)}{g(x)} = 1$. When it is clear from the context, we omit a in $\overset{a}{\approx}$ for convenience. the function $[x]^+$ is equal to x if x is positive and 0 otherwise. $\mathbb{R}_+^M$ designates the M-dimensional set of non-negative real numbers. For a non-negative integer n, $n!$ represents factorial n. $\text{Prob}(\mathcal{A})$ is the probability of the event $\mathcal{A}$. $F^{(n)}(x)$ represent the n-derivative of the function $F(\cdot)$ at x. Bold face letters represent M-dimension vectors, i.e., $\boldsymbol{x} = (x_1, \ldots, x_M)$.

and infinitely differentiable at 0 to invoke Taylor's theorem regarding expansion of $F_m(\cdot)$ around 0. We assume that N is sufficiently large ($N \to \infty$) and study performance limits of delay-limited communications at a constant rate R, for a given finite number of blocks M. Furthermore, the transmitter is constrained by a short term power constraint (STPC) defined by [17]: $\frac{1}{M}\sum_{m=1}^{M}\mu_m \le P$, where $\mu_m = \mathbf{E}[|x(m,n)|]^2$ for all $n = 1,\ldots,N$. The maximum mutual information of this channel is given by [17]: $I_M(\gamma,\boldsymbol{\mu}) = \frac{1}{M}\sum_{m=1}^{M}\frac{1}{2}\log(1+\gamma_m\,\mu_m)$ .. The outage probability computed at a rate R is given by:

$$P_{out}^{st,CSI-TR}(R,P,M) = \text{Prob}\{I_M(\gamma,\boldsymbol{\mu}) < R\}, \tag{2}$$

where the acronyms "st" stands for short therm. As in the sequel, we discuss performances when only CSI-R is available; we also use the acronyms CSI-TR and CSI-R as superscripts on the outage probability to distinguish between the two cases. In case of CSI-TR, $\boldsymbol{\mu}$ depends on γ, i.e., $\boldsymbol{\mu} = \boldsymbol{\mu}(\gamma)$. Then, it is of interest to find the optimal power policy that minimizes the outage probability given by (2).

The minimum outage probability of the BF-AWGN channel has been solved (even in a broader scope) in [17]. We outline briefly the related solution. Let the region Q be defined as $Q = \left\{\gamma_h \in \mathbb{R}_+^M :\ \gamma_1 \ge \ldots \ge \gamma_M\right\}$. Assume that $\gamma \in Q$, then the optimal power over the m-th block, $m = 1,\ldots,M$, is given by:

$$\mu_m(\gamma) = \left[\lambda(b,\gamma) - \frac{1}{\gamma_m}\right]^+, \tag{3}$$

where $\lambda(b,\gamma) = \frac{1}{b}\sum_{l=1}^{b}\frac{1}{\gamma_l} + \frac{M}{b}P$ and b is the unique integer in $\{1,\ldots,M\}$ such that $1/\gamma_m \le \lambda(b,\gamma)$ for $m < b+1$ and $1/\gamma_m > \lambda(b,\gamma)$ for $m \ge b+1$. In the previous formulation, b represents the number of blocks for which a positive instantaneous power is allocated. The optimal outage region $\mathcal{U}(R,P)$ is derived similarly by partitioning Q into M sub-regions $\mathcal{V}_m \subset Q$ such that if $\gamma \in \mathcal{V}_m$, exactly m blocks are given positive power. The intersection of $\mathcal{U}(R,P)$ with $\mathcal{V}_m$, $m = 1,\ldots,M$, is given explicitly by:

$$\mathcal{U}(R,P) \cap \mathcal{V}_m = \left\{\gamma \in Q : \frac{1}{M}\sum_{l=1}^{m}\frac{1}{2}\log(\gamma_l\,\lambda(m,\gamma)) < R\right\}. \tag{4}$$

To extend the optimal power and the outage region given by (3) and (4), respectively, beyond Q to $\mathbb{R}_+^M$, one may sort the component of γ in a non-increasing order, apply the above result on the sorted fading vector and apply the inverse permutation to the obtained power-allocation vector and outage probability region.

III. Performance Limits at Low-Power Regime Under A Short-Term Power Constraint

From the previous section, it is clear that beyond $M = 2$, there is generally no hope to derive the outage probability in a simple form thus preventing any insight on the behavior of the outage performance with respect to system parameters. This constitutes another motivation for us to look at the outage performance in the low-power regime.

The low-power regime is generally captured by letting $P \to 0$. In delay-unconstrained communication, this is enough since the reliability (arbitrary low decoding probability) is intrinsically contained in the definition of the channel capacity. However, for delay-constrained communications, since the capacity (in Shannon sense) is generally equal to zero,[2] studying the low-power regime can only be meaningful if the targeted outage performance is low in order to guarantee certain reliability. A simple manipulation of (2) provides:

$$\begin{aligned} P_{out}^{st,CSI-TR}(R,P,M) &\ge \text{Prob}\left\{\frac{1}{2}\log\left(1+\frac{1}{M}\sum_{m=1}^{M}\gamma_m\,\mu_m\right) < R\right\} && (5)\\ &\ge \text{Prob}\left\{\frac{1}{2}\log\left(1+\frac{1}{M}\sum_{m=1}^{M}\gamma_{max}\,\mu_m\right) < R\right\} && (6)\\ &\ge \text{Prob}\left\{\frac{1}{2}\log(1+\gamma_{max}\,P) < R\right\} && (7)\\ &\ge \text{Prob}\left\{\frac{1}{2}\gamma_{max}\,P < R\right\} && (8)\\ &= F_{max}\left(\frac{2R}{P}\right), && (9)\end{aligned}$$

where (5) follows because $\frac{1}{M}\sum_{m=1}^{M}\frac{1}{2}\log(1+\gamma_m\,\mu_m) \le \frac{1}{2}\log\left(1+\frac{1}{M}\sum_{m=1}^{M}\gamma_m\,\mu_m\right)$ due to the Jensen's inequality; (6) holds since $\gamma_{max} = \max_{m=1}^{M}\gamma_m$; (7) is a consequence of the power constraint; to obtain (8), we used the inequality $\log(1+x) \le x$ and finally $F_{max}(\cdot)$ in (9) is the CDF of γ_{max}. From (9), it is clear that if $P \to 0$ and R is arbitrary (not necessarily small), then the outage probability is arbitrary close to 1. Therefore, in delay-constrained communications, reliability at low-power regime can only be obtained if $R \to 0$ too. Hence, the low-power regime we are interested in corresponds to the situation where P, R and the outage performance tend all toward 0. We are now ready to state our main result.

Theorem 1: For the block-fading channel described by (1), with perfect CSI-TR, and subject to a STPC, the outage performance is characterized as follows:

$$P_{out}^{st,CSI-TR}(R,P,M) \overset{P\to 0}{\approx} \prod_{m=1}^{M} F_m\left(\frac{2\,R}{P}\right). \tag{10}$$

Proof: We establish a lower bound on $P_{out}^{st,CSI-TR}(R,P,M)$ and show that this bound coincides with the outage performance of a repetition diversity with optimal short-term power allocation, as $P \to 0$. Let us first define the regions Q_i, $i = 1,\ldots,M!$, as $Q_i = \left\{\gamma_h \in \mathbb{R}_+^M :\ \gamma_{\Pi_i(1)} \ge \ldots \ge \gamma_{\Pi_i(M)}\right\}$, where $\Pi_i(\cdot)$ is a certain permutation in $\{1,\ldots,M\}$. By construction, Q_i's form a partition of $\mathbb{R}_+^M$. Let us also split each region Q_i into M subregions $\mathcal{V}_{i,j}$, $j = 1,\ldots,M$, such that each subregion $\mathcal{V}_{i,j}$ corresponds to channel realizations in Q_i where exactly j blocks are given positive instantaneous power. Note also that $\left\{\mathcal{V}_{i,j} : j = 1\ldots,M\right\}$ is a partition of Q_i. Then, the following inequalities hold true:

$$\begin{aligned} P_{out}^{st,CSI-TR}(R,P,M) &= \text{Prob}\left(\mathcal{U}(R,P)\bigcap\bigcup_{i=1}^{M!}Q_i\right) && (11)\\ &= \sum_{i=1}^{M!}\text{Prob}\left(\mathcal{U}(R,P)\bigcap Q_i\right) && (12)\end{aligned}$$

[2] This might not be the case for a class of fading if CSI-T is available. However, in that case the outage would be equal to zero anyway.

$$
\begin{aligned}
&= \sum_{i=1}^{M!}\text{Prob}\left(\mathcal{U}(R,P)\bigcap\left(\bigcup_{j=1}^{M}\mathcal{V}_{i,j}\right)\right) && (13)\\
&= \sum_{i=1}^{M!}\sum_{j=1}^{M}\text{Prob}\left(\mathcal{U}(R,P)\bigcap\mathcal{V}_{i,j}\right) && (14)\\
&\geq \sum_{i=1}^{M!}\text{Prob}\left(\mathcal{U}(R,P)\bigcap\mathcal{V}_{i,1}\right) && (15)\\
&= \sum_{i=1}^{M!}\text{Prob}\left(\gamma\in\mathcal{V}_{i,1};\gamma_{\Pi_i(1)}<x_1\right) && (16)\\
&= \sum_{i=1}^{M!}\text{Prob}\Big(\gamma_{\Pi_i(M)}\leq\ldots\leq\gamma_{\Pi_i(1)} \\
&\qquad \leq\frac{\gamma_{\Pi_i(1)}}{1+MP\,\gamma_{\Pi_i(1)}};\gamma_{\Pi_i(1)}<x_1\Big) && (17)\\
&\approx \sum_{i=1}^{M!}\text{Prob}\left(\gamma_{\Pi_i(M)}\leq\ldots\leq\gamma_{\Pi_i(1)}<x_1\right) && (18)\\
&= \sum_{i=1}^{M!}\text{Prob}\left(\gamma\in\mathcal{Q}_i;\max_{0\leq m\leq M-1}\gamma_m<x_1\right) && (19)\\
&= \text{Prob}\left(\left(\max_{1\leq m\leq M}\gamma_m<x_1\right)\bigcap\bigcup_{i=1}^{M!}\mathcal{Q}_i\right) && (20)\\
&= \text{Prob}\left(\max_{1\leq m\leq M}\gamma_m<x_1\right) && (21)\\
&= \prod_{m=1}^{M}F_m(x_1) && (22)\\
&\approx \prod_{m=1}^{M}F_m\left(\frac{2R}{P}\right) && (23)
\end{aligned}
$$

where $x_1 = \frac{e^{2MR}-1}{MP}$; (16) follows because since only 1 block is used for transmission, then the corresponding channel mutual information is equal to $\frac{1}{2M}\log\left(1+\left(\frac{1}{\gamma_{\Pi_i(0)}}+MP-\frac{1}{\gamma_{\Pi_i(0)}}\right)\gamma_{\Pi_i(0)}\right)$; (17) follows similarly because only 1 block is used for transmission and thus $\frac{1}{\gamma_{\Pi_i(0)}}+MP<\frac{1}{\gamma_{\Pi_i(m)}}$ for all $m=1,\ldots,M-1$; (18) is justified by the fact that $\frac{\gamma_{\Pi_i(1)}}{1+MP\gamma_{\Pi_i(1)}}\approx\gamma_{\Pi_i(1)}$ as $P\to0$, and finally (23) holds true because since $P_{out}^{st,CSI-TR}(R,P,M)\to0$ and $P\to0$, then so does R and thus x_1 in Theorem 1 is asymptotically equal to $\frac{2R}{P}$. The lower bound (22) is exactly the outage probability of a selection diversity that allocates to a block m, $m=1,\ldots,M$, a power P_m such that

$$
P_m=\begin{cases} MP & \text{if} \quad \gamma_m=\max\limits_{1\leq k\leq M}\gamma_k\\ 0 & \text{otherwise.}\end{cases} \tag{24}
$$

Hence the lower bound is asymptotically achievable and Theorem 1 is thus proved. ■

Few remarks are worthwhile.

Remark 1: For the class of fading such that $F_m(x)>0$ for all $x>0$, $m=1,\ldots,M$ (this is the case of most popular fading in wireless communications), since $P_{out}^{st,CSI-TR}(R,P,M)\to0$, then Theorem 1 implies that we necessarily have $\frac{R}{P}\to0$. That is, for this class of fading, reliable communication (communication with arbitrary low outage probability) at low-power regime requires not only that the rate R be small ($R\to0$), but more stringently that the ratio $\frac{R}{P}$ be small too ($\frac{R}{P}\to0$).

Remark 2: While it is well-known that increasing M can only decrease the outage performance (assuming optimal coding), Theorem 1 describes how the outage probability decreases with M. In fact, if the number of blocks over which communication is happening increases from M to $M+1$, the outage probability decreases by a factor of $F_{M+1}\left(\frac{2R}{P}\right)$. More specifically, one can express the outage probability in a more explicit way as follows:[3],

$$
\begin{aligned}
\log\left(P_{out}^{st,CSI-TR}(R,P,M)\right) &\approx \log\left(\prod_{m=1}^{M}F_m\left(\frac{2R}{P}\right)\right) && (25)\\
&\approx \log\left(\prod_{m=1}^{M}\frac{F_m^{(n_m)}(0)}{n_m!}\left(\frac{2R}{P}\right)^{n_m}\right) && (26)\\
&\approx \log\left(\frac{2R}{P}\right)\sum_{m=1}^{M}n_m, && (27)\\
&= -\log\left(\frac{P}{2R}\right)\sum_{m=1}^{M}n_m && (28)
\end{aligned}
$$

where n_m is the smallest integer such that $F_m^{(n_m)}(0)\neq0$, $m=1,\ldots,M$; (26) is obtained by series expansion of $F_m(x)$ around zero and because $\frac{R}{P}\to0$ (see *Remark* 1) and (27) follows because $\lim_{\frac{R}{P}\to0}\frac{\log\left(\frac{F_m^{(n_m)}(0)}{n_m!}\right)}{n_m\log\left(\frac{2R}{P}\right)}=0$. For fixed P and R, (28) describes how does the outage probability depends on M. Note that since $n_m\geq1$, for all $m=1,\ldots,M$, then $\sum_{m=1}^{M}n_m\geq M$. Also, because $\frac{P}{2R}\to\infty$, then $\sum_{m=1}^{M}n_m$ may be regarded as the diversity order provided by coding over M blocks at low-power regime.

Remark 3: The result in Theorem 1 translates directly to a characterization of the so-called outage capacity. Indeed, along similar steps as *Remark* 2, it is easy to verify that the outage capacity $C_\epsilon(\cdot)$ is given by:

$$
C_\epsilon(P,M)\approx\frac{1}{2}\left(\prod_{m=1}^{M}\frac{n_m!}{F_m^{(n_m)}(0)}\right)^{\frac{1}{\sum_{m=0}^{M-1}n_m}}\epsilon^{\frac{1}{\sum_{m=0}^{M-1}n_m}}P. \tag{29}
$$

The ratio $\frac{C_\epsilon(P,M)}{P}$, defined as the capacity per unit power for a given outage probability ϵ, is a key performance measure in the regime of interest [15]. Therefore, our result also characterizes the capacity per unit power through (29).

We conclude this section by mentioning that although at low-power regime channel estimation might be challenging, the optimal scheme given by (24) does not perform power adaptation and hence only requires few feedback bits in each frame of M blocks to designate the block with the highest channel gain. More precisely, it needs exactly $\lceil\log_2(M)\rceil$ feedback bits, where $\lceil\cdot\rceil$ is the ceiling function. A question of interest in this case would be to assess whether the additional gain provided by CSI-TR over CSI-R is worth the above low rate feedback mechanism. This is answered in Section IV.

[3]Note that we are computing the outage probability at the log scale in order to show the diversity order. Note also that although our focus is on low-power regime, the diversity order is still meaningful since the outage probability is arbitrary driven to zero as a consequence of the fact that $\frac{R}{P}\to0$.

IV. How Rewarding Perfect CSI At The transmitter is?

We first evaluate the outage probability assuming perfect CSI-R and no CSI-T. Recall that with CSI-R and under a STPC, the outage probability (using constant power over the blocks) is given by:

$$P_{out}^{st,CSI-R}(R,P,M) = \text{Prob}\left(\frac{1}{M}\sum_{m=1}^{M}\frac{1}{2}\log(1+P\,\gamma_m) < R\right) \quad (30)$$

$$\approx \text{Prob}\left(\sum_{m=1}^{M}\gamma_m < \frac{2\,M\,R}{P}\right), \quad (31)$$

$$= F_{\Sigma}\left(\frac{2\,M\,R}{P}\right), \quad (32)$$

where (31) follows because $\log(1+x) \approx x$ around zero, and where $F_{\Sigma}(\cdot)$ is the CDF of $\sum_{m=1}^{M}\gamma_m$. Note that the right hand side (RHS) of (31) is asymptotically equal to the outage probability of a repetition diversity with constant-power transmission. Hence, with CSI-R, repetition diversity with constant-power transmission is an optimal strategy at asymptotically low-power regime.

There is apparently no explicit expression of $F_{\Sigma}(\cdot)$ for general fading distributions of γ_m, $m=1,\ldots,M$. Nevertheless, we have the following result that characterizes the gain in terms of outage performance due to CSI-TR.

Theorem 2: Let n_m be the smallest integer such that $F_m^{(n_m)}(0) \neq 0$, $m=1,\ldots,M$. Then, we have:

$$\lim_{P\to 0}\frac{P_{out}^{st,CSI-TR}(R,P,M)}{P_{out}^{st,CSI-R}(R,P,M)} = \frac{1}{M^{\sum_{m=1}^{M}n_m}}\frac{\left(\sum_{m=1}^{M}n_m\right)!}{\prod_{m=1}^{M}n_m!}. \quad (33)$$

Proof: For convenience, the proof is presented in Appendix A ■

Since the availability of CSI-T can only decrease the outage performance $P_{out}^{st,CSI-R}(R,P,M)$, one needs to verify that the right hand side (RHS) of (33) is at most equal to 1. This is not very clear at first glance. However, we show below that this is in fact a direct consequence of the following lemma that provides a more general result.

Lemma 1: Let n be a positive integer. Let V_n be the simplex defined by: $V_n = \left\{(t_1,\ldots,t_n)\in[0,\infty)^n : \sum_{k=1}^{n}t_k \leq 1\right\}$. Then, for any $(a_1,\ldots,a_n)\in(0,\infty)^n$, we have:

$$\frac{\Gamma(1+a_1+\cdots+a_n)}{\Gamma(1+a_1)\cdots\Gamma(1+a_n)} \leq n^{\sum_{k=1}^{n}a_k}, \quad (34)$$

where $\Gamma(\cdot)$ is the Gamma function defined by: $\Gamma(x) = \int_0^{\infty}e^{-t}\,t^{x-1}\mathrm{d}t$, for $x>0$.

Proof: We first recall a multidimensional integral property of the Gamma function [18]:

$$\int_{V_n}t_1^{a_1-1}\cdots t_n^{a_n-1}\mathrm{d}t_1\cdots\mathrm{d}t_n = \frac{\Gamma(a_1)\cdots\Gamma(a_n)}{\Gamma(1+a_1+\cdots+a_n)} \quad (35)$$

Then, we need to lower bound the left hand side (LHS) of (35). For this purpose, we let X_k be a random variable with PDF $f_{X_k}(t) = a_k\,t^{a_k-1}$, for $t\in[0,1]$, $k=1,\ldots,n$.[4] We also assume that X_k's are jointly independent such that their joint PDF $f_{(X_1,\ldots,X_n)}(t_1,\ldots,t_n) = \prod_{k=1}^{n}f_{X_k}(t_k)$. The LHS of (35) can be lower bounded as follows:

$$\int_{V_n}t_1^{a_1-1}\cdots t_n^{a_n-1}\mathrm{d}t_1\cdots\mathrm{d}t_n$$

$$= \frac{1}{\prod_{k=1}^{n}a_k}\int_{V_n}\prod_{k=1}^{n}f_{X_k}(t_k)\,\mathrm{d}t_1\cdots\mathrm{d}t_n \quad (36)$$

$$= \frac{1}{\prod_{k=1}^{n}a_k}\int_{V_n}f_{(X_1,\ldots,X_n)}(t_1,\ldots,t_n)\,\mathrm{d}t_1\cdots\mathrm{d}t_n \quad (37)$$

$$= \frac{1}{\prod_{k=1}^{n}a_k}\,\text{Prob}\left\{\sum_{k=1}^{n}X_k \leq 1\right\} \quad (38)$$

$$\geq \frac{1}{\prod_{k=1}^{n}a_k}\,\text{Prob}\left\{X_k \leq \frac{1}{n},\; k=1,\ldots,n\right\} \quad (39)$$

$$= \frac{1}{\prod_{k=1}^{n}a_k\,n^{a_k}}. \quad (40)$$

Since for any $x>0$, $\Gamma(1+x) = x\,\Gamma(x)$, (34) follows from (35) and (40) immediately. ■

Finally, we note that the RHS of (33) is indeed smaller than 1 because:

$$\frac{1}{M^{\sum_{m=1}^{M}n_m}}\frac{\left(\sum_{m=1}^{M}n_m\right)!}{\prod_{m=1}^{M}n_m!} = \frac{1}{M^{\sum_{m=1}^{M}n_m}}\frac{\Gamma(1+n_1+\ldots+n_M)}{\Gamma(1+n_1)\ldots\Gamma(1+n_M)} \leq 1, \quad (41)$$

where the last inequality follows from Lemma 1. There is no particular difficulty in evaluating the RHS of (33) for given fading channels. For example, for Rayleigh fading, $n_m=1$, $m=1,\ldots,M$, and the RHS of (33) is equal to $\frac{M!}{M^M}$. The later result holds for any fading channel that satisfies $n_m=1$, $m=0,\ldots,M-1$. Since $\frac{M!}{M^M}$ is strictly decreasing in M,[5] then it follows from Theorem 2 that for this class of fading, increasing M strictly increases the gap between $P_{out}^{st,CSI-TR}(R,P,M)$ and $P_{out}^{st,CSI-R}(R,P,M)$. More generally, for all fading channels such that n_m is a constant, $m=1,\ldots,M$, we have the following.

Corollary 1: Let n_m be the smallest integer such that $F_m^{(n_m)}(0)\neq 0$. Then, if $\forall m=1,\ldots,M$, $n_m=n$, then we have:

i.
$$\lim_{P\to 0}\frac{P_{out}^{st,CSI-TR}(R,P,M)}{P_{out}^{st,CSI-R}(R,P,M)} = \frac{(M\,n)!}{M^{n\,M}\;(n!)^M}, \quad (42)$$

ii. the ratio $\frac{(M\,n)!}{M^{n\,M}\,(n!)^M}$ is strictly decreasing in M, for any $n\geq 1$ and thus increasing M strictly enhances the advantage of CSI-TR over CSI-R, in term of outage probability,

iii. the ratio $\frac{(M\,n)!}{M^{n\,M}\,(n!)^M}$ converges to 0 as $M\to\infty$ and hence the advantage of CSI-TR over CSI-R is unbounded.

Proof: Property i in Corollary 1 is an immediate consequence of Theorem 2 and the fact that $n_m=n$, for all $m=0,\ldots,M-1$. To prove ii, we recall first the Stirling's formula [19]:

$$1 \leq \frac{k!}{\sqrt{2\,\pi\,k}\,\left(\frac{k}{e}\right)^k} \leq \frac{e}{\sqrt{2\,\pi}}, \quad (43)$$

for any integers $k\geq 1$. Then, we let $U(M) = \frac{(M\,n)!}{M^{n\,M}\,(n!)^M}$ and use (43) to upper bound $U(M)$ as follows:

$$1 \leq \frac{U(M)}{\left(\frac{n}{e}\right)^{Mn}\frac{\sqrt{2\pi Mn}}{(n!)^M}} \leq \frac{e}{\sqrt{2\,\pi}}. \quad (44)$$

[4] Note that if for some k, $a_k\in(0,1)$, then the support of X_k can be restricted to $(0,1]$ instead of $[0,1]$, without any consequence on the proof.

[5] This is because $\frac{(M+1)!}{(M+1)^{M+1}}\,\frac{M^M}{M!} = \left(\frac{M}{M+1}\right)^M \leq 1$.

Now, with the help of (44), one can upper bound $\frac{U(M+1)}{U(M)}$ as follows:

$$\frac{U(M+1)}{U(M)} \leq \frac{e}{\sqrt{2\pi}}\left(\frac{n}{e}\right)^{(M+1)n}\frac{\sqrt{2\pi(M+1)n}}{(n!)^{(M+1)}}\frac{1}{\left(\frac{n}{e}\right)^{Mn}\frac{\sqrt{2\pi Mn}}{(n!)^{M}}} \tag{45}$$

$$= \frac{e}{\sqrt{2\pi}}\left(\frac{n}{e}\right)^{n}\sqrt{\frac{M+1}{M}}\frac{1}{n!} \tag{46}$$

$$\leq \frac{e}{\sqrt{2\pi}}\sqrt{\frac{M+1}{M}}\frac{1}{\sqrt{2\pi n}} \tag{47}$$

$$\leq \frac{e}{2\pi}\sqrt{\frac{M+1}{M}} \tag{48}$$

$$\leq \frac{e}{2\pi}\sqrt{2} \tag{49}$$

$$< 1, \tag{50}$$

where (47) is obtained using the LHS of (43); (48) holds because $n \geq 1$ and (49) follows from the fact that the function $x \mapsto \sqrt{\frac{x+1}{x}}$ is monotonically decreasing and thus for all $M \geq 1$, $\sqrt{\frac{M+1}{M}} \leq \sqrt{\frac{1+1}{1}} = \sqrt{2}$. Therefore, $U(M)$ is strictly increasing in M and property ii is thus proved. Finally, in order to verify iii, we again use (44) and observe that:

$$U(M) \leq \frac{e}{\sqrt{2\pi}}\left(\frac{n}{e}\right)^{Mn}\frac{\sqrt{2\pi Mn}}{(n!)^{M}} \tag{51}$$

$$= e\left(\frac{\left(\frac{n}{e}\right)^{n}}{n!}\right)^{M}\sqrt{n}\,\sqrt{M} \tag{52}$$

$$\leq e\left(\frac{1}{\sqrt{2\pi n}}\right)^{M}\sqrt{n}\,\sqrt{M}, \tag{53}$$

where (53) follows from the LHS of (43). Clearly, the RHS of (53) converges to zero as $M \to \infty$ and since $U(M)$ is non-negative, then $\lim_{M\to\infty} U(M) = 0$. Hence, property iii holds true and Corollary 1 is proved. ∎

Note that Corollary 1 encompasses the important particular case where the fading over the M blocks are identically distributed since in that case n_m would be constant for all $m = 1, \ldots, M$. Furthermore, in light of Corollary 1, it can be inferred that the advantage of CSI-TR over CSI-R can be tremendous for M sufficiently large, at low-power regime, provided that the targeted outage performance (or equivalently the rate R at which we communicate) is low. For instance, using (10), the power required to achieve an outage performance ϵ while communicating at a rate R assuming CSI-TR is asymptotically equal to:

$$P^{st,CSI-TR}(\epsilon, R) \approx \frac{2R}{\left(\prod_{m=1}^{M}\frac{n_m!}{F_m^{(n_m)}(0)}\right)^{\frac{1}{\sum_{m=1}^{M} n_m}}\epsilon^{\frac{1}{\sum_{m=1}^{M} n_m}}}. \tag{54}$$

Whereas with CSI-R only, the required power is equal to:[6]

$$P^{st,CSI-R}(\epsilon, R) \approx \frac{2MR}{\left(\frac{\left(\sum_{m=1}^{M} n_m\right)!}{\prod_{m=1}^{M}F_m^{(n_m)}(0)}\right)^{\frac{1}{\sum_{m=1}^{M} n_m}}\epsilon^{\frac{1}{\sum_{m=1}^{M} n_m}}}. \tag{55}$$

[6]The derivation of (55) follows from (85) in the Appendix.

Hence, the power gain due to CSI-TR is equal to:

$$\frac{P^{st,CSI-TR}(\epsilon, R)}{P^{st,CSI-R}(\epsilon, R)} \approx \left(\frac{\left(\sum_{m=1}^{M} n_m\right)!}{M^{\sum_{m=1}^{M} n_m}\prod_{m=1}^{M} n_m!}\right)^{\frac{1}{\sum_{m=1}^{M} n_m}}. \tag{56}$$

Now, if $M = 2$ and $n_m = 1$, for all $m = 1, \ldots, M$, (56) yields 1.5 dB gain which is considerable in the low-power regime. Increasing M to 3 enhances further the gain to about 2.18 dB. Actually, it can be easily shown that as $M \to \infty$, the ratio $\frac{P^{st,CSI-TR}(\epsilon,R)}{P^{st,CSI-R}(\epsilon,R)}$ converges to e^{-1} and hence for M sufficiently large and assuming all $n_m = 1$, a remarkable 4.3 dB gain is achievable. Note that there is no contradiction between the fact that the advantage of CSI-TR over CSI-R in terms of outage probability is unbounded as stated by iii in Corollary 1 and the fact that the power gain resulting from CSI-T is bounded. This is because the exponent in (56) also depends on M. Therefore, the common belief by which "optimal power allocation under the short-term constraint does not provide any significant gain with respect to constant power" is somehow misleading. In [17], the authors pointed out that for Rayleigh fading channels, "As M increases, the outage probability with short-term constraint is always very close to that for constant power". This guideline is unfortunately not very precise and hence should be looked at more carefully, at least in the low-power regime. Indeed, as stated by Corollary 1, the outage probability with the short-term constraint can be arbitrary smaller than that of constant power, for large M.

So far, we have focused on the case where all n_m are equal, i.e., $n_m = n$ is constant and fixed, and we have analyzed the value of CSI-TR over CSI-R in function of M. Another aspect of interest would be to fix M, and study the reward provided by CSI-TR versus n. This would help understanding the value of CSI-TR for different fading channels, for a given delay constraint. For convenience, our result is summarized in the following corollary.

Corollary 2: Let n_m be the smallest integer such that $F_m^{(n_m)}(0) \neq 0$, $m = 1, \ldots, M$. Then, if $\forall m = 1, \ldots, M$, $n_m = n$, then we have:

i. the ratio $\frac{(Mn)!}{M^{nM}(n!)^M}$ is strictly decreasing in n, for any $M > 1$, and thus the higher n is, the more important is the advantage of CSI-TR over CSI-R, in term of outage probability,

ii.
$$\lim_{n\to\infty}\frac{(Mn)!}{M^{nM}(n!)^{M}} = \begin{cases} 0, & \text{if } M > 1 \\ 1, & \text{if } M = 1. \end{cases} \tag{57}$$

Proof: First, we note that if $M = 1$, the ratio in Corollary 2 is equal to 1, irrespective of n. Now, assume that $M > 1$ and let $V(n) = \frac{(Mn)!}{M^{nM}(n!)^M}$. Since the bounds in (44) hold for $V(n)$, then we can upper bound $\frac{V(n+1)}{V(n)}$ as follows:

$$\frac{V(n+1)}{V(n)} \leq \frac{e}{\sqrt{2\pi}}\left(\frac{n+1}{e}\right)^{M(n+1)}\frac{\sqrt{2\pi M(n+1)}}{((n+1)!)^{M}}\frac{1}{\left(\frac{n}{e}\right)^{Mn}\frac{\sqrt{2\pi Mn}}{(n!)^{M}}} \tag{58}$$

$$= \frac{e}{\sqrt{2\pi}}\frac{1}{e^{M}}\sqrt{\frac{1+n}{n}}\left(\frac{1+n}{n}\right)^{Mn} \tag{59}$$

$$\leq \frac{e}{\sqrt{2\pi}}\frac{1}{e^{M}}\sqrt{2}\,2^{M} \tag{60}$$

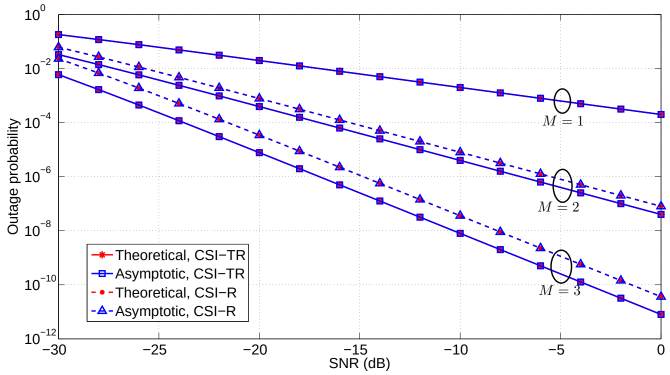

Figure 1. Outage probability versus SNR for Rayleigh fading channels, for different M values, and $R = 10^{-4}$ npcu, at low-power regime.

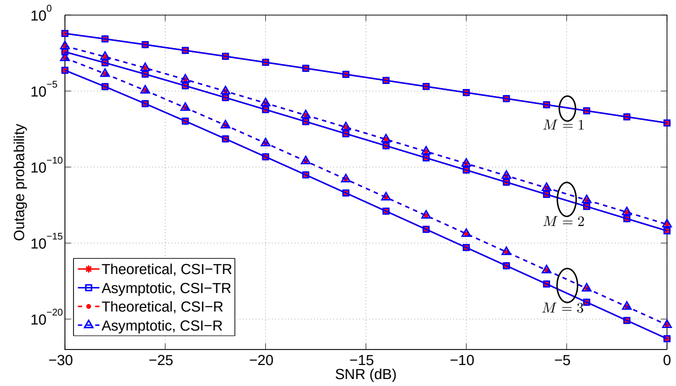

Figure 2. Outage probability versus SNR for Nakagami-m fading channels ($m = 2$), for different M values, and $R = 10^{-4}$ npcu, at low-power regime.

$$= \frac{e}{\sqrt{2}\,\pi} e^{\left(M+\frac{1}{2}\right)\log(2)-M} \tag{61}$$
$$\leq \frac{e}{\sqrt{2}\,\pi} e^{\left(2+\frac{1}{2}\right)\log(2)-2} \tag{62}$$
$$= \frac{1}{\sqrt{\pi}}\frac{4}{e} \tag{63}$$
$$< 1, \tag{64}$$

where (60) follows because the function $x \mapsto \sqrt{\frac{1+x}{x}}\left(\frac{1+x}{x}\right)^{Mx}$ is the product of two positive-definite monotonically decreasing functions and thus is also monotonically decreasing; (62) is true because the function $x \mapsto \left(x+\frac{1}{2}\right)\log(2) - x$ is monotonically decreasing and also because $M \geq 2$. Hence $V(n)$ is strictly decreasing in n. To prove ii, we upper bound $V(n)$ departing from (51) as follows:

$$V(n) \leq \frac{e}{\sqrt{2}\,\pi}\left(\frac{n}{e}\right)^{Mn}\frac{\sqrt{2\pi Mn}}{(n!)^M} \tag{65}$$
$$\leq \frac{e}{\sqrt{2}\,\pi}\left(\frac{n}{e}\right)^{Mn}\frac{\sqrt{2\pi Mn}}{\left(\sqrt{2\pi n}\left(\frac{n}{e}\right)^n\right)^M} \tag{66}$$
$$= \frac{e}{\sqrt{2}\,\pi}\frac{\sqrt{M}}{(2\pi)^{\frac{M-1}{2}}}\frac{1}{n^{\frac{1}{2}(M-1)}}, \tag{67}$$

where (66) is due to the LHS of (43). Since the RHS of (67) converges to 0 as $n \to \infty$ for any $M > 1$, then so does $V(n)$. This completes the proof of Corollary 2. ■

V. Numerical Results

We present results for normalized i.i.d. Rayleigh and Nakagami-m ($m = 2$) fading channels in Fig. 1 and 2, respectively. Figure 1 shows the outage probability versus P for a given R and for $M = 1$, $M = 2$ and $M = 3$. For each M, Fig. 1 shows the curves corresponding to the exact outage under CSI-TR, its asymptotic expression in Theorem 1, the exact outage under CSI-R and its asymptotic expression given by (30) and (32), respectively. The exact outage under CSI-TR has been obtained by numerical integration of the channel PDF over the outage region given by (4). For $M = 1$, the outage probability under CSI-TR and CSI-R always coincide, thus we only display the curves corresponding to CSI-TR in this case. Figure 1 shows the accuracy of our asymptotic analysis since the exact and the asymptotic representations match perfectly. As M increases, the ratio between the outage probabilities under CSI-TR and CSI-R decreases as predicted by Corollary 1. Also, for a given outage performance, the power gain due to CSI-T increases form 0 dB for $M = 1$ to about 1.5 dB for $M = 2$ and reaches 2.18 dB for $M = 3$, exactly as predicted by (56). Finally, we note that the slopes of the curves in Fig. 1 are equal to 1, 2 and 3 for $M = 1$, $M = 2$ and $M = 3$, respectively, since for Rayleigh fading, $n_m = 1$, as discussed in Remark 2. For Nakagami-m fading, in addition to the above observations, we note first that the slopes of the curves in Fig. 2 are equal to 2, 4 and 6 for $M = 1$, $M = 2$ and $M = 3$, respectively, since for Nakagami-m ($m = 2$) fading, $n_m = 2$. Comparing the impact of the two fading on the outage, we observe that for $M = 3$ and $P = -4$ dB for instance, the ratio between the outage probabilities with CSI-TR and with CSI-R is equal to about 0.22 in Fig. 1 whereas it is equal to 0.12 in Fig. 2 implying that the advantage of CSI-TR increases with n_m, again in agreement with Corollary 2.

VI. Conclusion

We have studied performance limits of delay-constrained communications with perfect CSI-TR at low-power regime. We have shown that selection diversity achieves asymptotically the minimum outage performance, and provided a simple expression of the outage probability. We have highlighted the gain of CSI-TR over CSI-R only, in terms of outage probability, and shown that this gain grows unboundedly with the delay constraint. Finally, we have discussed the value of CSI-TR for different fading channels in the low-power regime.

Appendix A
Proof of Theorem 2

We designate by $\mathcal{L}\{f\}$ the Laplace transform of the function f. Let $\phi_m = \mathcal{L}\{f_m\}$, $m = 1, \ldots, M$. Since n_m be the smallest integer such that $F_m^{(n_m)}(0) \neq 0$, then there exist $\eta_m > 0$, such that for any $x \in (0, \eta_m)$, we have:

$$F_m(x) = \frac{F_m^{(n_m)}(0)}{n_m!} x^{n_m} + \mathrm{o}\left(x^{n_m}\right), \tag{68}$$

$m = 1, \ldots, M$. Thus, for any $x \in (0, \min_{m=1}^{M} \eta_m)$, we have:

$$\prod_{m=1}^{M} F_m(x) \approx \prod_{m=1}^{M} \frac{F_m^{(n_m)}(0)}{n_m!} x^{n_m}. \tag{69}$$

Hence, $P_{out}^{st,CSI-TR}(R,P,M)$ is asymptotically equal to:

$$P_{out}^{st,CSI-TR}(R,P,M) \approx \prod_{m=1}^{M} F_m\left(\frac{2R}{P}\right) \tag{70}$$
$$\approx \prod_{m=1}^{M} \frac{1}{M^{n_m}} \frac{F_m^{(n_m)}(0)}{n_m!}\left(\frac{2MR}{P}\right)^{n_m}, \tag{71}$$

where (71) is a consequence of (69) and the fact that as $P_{out}^{st,CSI-TR}(R,P,M) \to 0$, then we necessarily have $\frac{R}{P} \to 0$. Before computing $P_{out}^{st,CSI-R}(R,P,M)$, we note first that since $F_m^{(n_m)}(0) \neq 0$, then we have:

$$0 \neq F_m^{(n_m)}(0) \tag{72}$$
$$= \lim_{x\to 0} F_m^{(n_m)}(x) \tag{73}$$
$$= \lim_{s\to\infty} s\,\mathcal{L}\left\{F_m^{(n_m)}\right\}(s) \tag{74}$$
$$= \lim_{s\to\infty} s\left(s^{n_m}\,\mathcal{L}\{F_m\}(s) - \sum_{k=1}^{n_m} s^{k-1} F_m^{(n_m-k)}(0)\right) \tag{75}$$
$$= \lim_{s\to\infty} s\, s^{n_m}\,\frac{1}{s}\,\mathcal{L}\{f_m\}(s) \tag{76}$$
$$= \lim_{s\to\infty} s^{n_m}\,\phi_m(s\}, \tag{77}$$

where (74) follows from the Initial Value Theorem; (75) holds true by applying the Laplace transform rule of the n_m–th derivative and (76) is due to the fact that n_m is the smallest integer such that $F_m^{(n_m)}(0) \neq 0$ and also by applying the Laplace transform rule of the integral. That is $\lim_{s\to\infty} s^{n_m}\,\phi_m(s\}$ exists and is non-null. We are now ready to compute $P_{out}^{st,CSI-R}(R,P,M)$. Let n be the smallest integer such that $F_\Sigma^{(n)}(0) \neq 0$, then again there exists $\eta > 0$, such that for any $x \in (0,\eta)$, we have:

$$F_\Sigma(x) = \frac{F_\Sigma^{(n)}(0)}{n!}\,x^n + \mathrm{o}(x^n). \tag{78}$$

We compute $F_\Sigma^{(n)}(0)$ as follows:

$$F_\Sigma^{(n)}(0) = \lim_{x\to 0} F_\Sigma^{(n)}(x) \tag{79}$$
$$= \lim_{s\to\infty} s^n\,\mathcal{L}\{f_\Sigma\}(s) \tag{80}$$
$$= \lim_{s\to\infty} s^n \prod_{m=1}^{M} \phi_m(s) \tag{81}$$
$$= \lim_{s\to\infty} \frac{s^n}{s^{\sum_{m=1}^{M} n_m}} \prod_{m=0}^{M-1} s^{n_m}\,\phi_m(s), \tag{82}$$

where (80) is justified exactly as (76), (81) follows because the PDF of $\sum_{m=1}^{M}\gamma_m$ is given by $f_\Sigma(x) = (f_1 * \ldots * f_M)(x)$, where $*$ designate the convolution operator; and (82) holds true since both limits in (77) and (81) exist. Thus, the limit in (82) exists and is non-null (since $F_\Sigma^{(n)}(0) \neq 0$). But, $\lim_{s\to\infty} \prod_{m=0}^{M-1} s^{n_m}\,\phi_m(s) \neq 0$ due to (77). Hence $\lim_{s\to\infty} \frac{s^n}{s^{\sum_{m=1}^{M} n_m}} \neq 0$ which implies that $n = \sum_{m=1}^{M} n_m$. Consequently, $F_\Sigma^{(n)}(0)$ can be further simplified as:

$$F_\Sigma^{(n)}(0) = \lim_{s\to\infty} \prod_{m=1}^{M} s^{n_m}\,\phi_m(s) \tag{83}$$
$$= \prod_{m=1}^{M} F_m^{(n_m)}(0), \tag{84}$$

where (84) is obtained from (83) using both (77) and (72). Combining (32), (78) and (84), the outage probability with CSI at the receiver can be expressed by:

$$P_{out}^{st,CSI-R}(R,P,M) \approx \frac{\prod_{m=1}^{M} F_m^{(n_m)}(0)}{\left(\sum_{m=1}^{M} n_m\right)!}\left(\frac{2MR}{P}\right)^{\sum_{m=1}^{M} n_m}. \tag{85}$$

Finally, using (71) and (85), we have:

$$\frac{P_{out}^{st,CSI-TR}(R,P,M)}{P_{out}^{st,CSI-R}(R,P,M)} \approx \frac{1}{M^{\sum_{m=1}^{M} n_m}}\,\frac{\left(\sum_{m=1}^{M} n_m\right)!}{\prod_{m=1}^{M} n_m!}, \tag{86}$$

which we wanted to prove.

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