Performance Analysis of Communications under Energy Harvesting Constraints with Noisy Channel State Information

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***Abstract*—In energy harvesting communications, the transmitters have to adapt transmission to availability of energy harvested during the course of communication. The performance of the transmission depends on the channel conditions which vary randomly due to mobility and environmental changes. In this paper, we consider the problem of power allocation taking into account the energy arrivals over time and the degree of channel state information (CSI) available at the transmitter, in order to maximize the throughput. Differently from previous work, the CSI at the transmitter is not perfect and may include estimation errors. We solve this problem with respect to the causality and energy storage constraints. We determine the optimal power policy in the case where the channel is assumed to be perfectly known at the receiver. Furthermore, a study of the asymptotic behavior of the communication system is proposed, we analyze of the average throughput (AT) in a system where the average recharge rate (ARR) goes asymptotically to zero and when it is very high. Numerical results provide insight into the performance of energy harvesting system in terms of data that can be sent.**

***Index Terms*—Asymptotic average throughput, channel estimation, channel state information, energy harvesting, optimal power policy, throughput maximization.**

I. Introduction

In the light of the massive developments in information and communication technologies (ICT), the energy consumption related to ICT is expected to grow at an alarming rate. Moreover, the expansion of information technology (IT) in mobile networks is growing continuously nowadays and this intense growth is likely to be continued for the next decades. Consequently, the use of the IT for communications networks is fundamental in order to improve the performance of these networks. Therefore, new challenges are faced the community of research to solve the problem of energy consumption. In this context, recent advances in communication and the massive search for green technologies are leading to propose a promising technology to prolong lifetime of wireless sensor networks (WSNs) [1]. Differently from the traditional wireless communication systems where communication devices are mainly powered by a replaceable or non-rechargeable batteries, the concept of energy harvesting (EH) was proposed to enhance the energy efficiency of a WSN. Alternatively, the use of EH technology is based on the replenishment of the energy from ambient sources such as solar energy, wind and vibrations, etc. The performance of a system equipped by this technology is determined by adapting the transmission power to energy availability [2]. The analysis of this performance leads to several transmission methods and power allocation policies that were presented in recent works.

A. Related work

As a starting point of EH communications, point-to-point data transmission have been studied in the literature, based on throughput maximization and time completion minimization. Several works were done to investigate the optimal power scheme [3]-[6] in the offline case, i.e., the transmitter (TX) has a perfect knowledge about the energy profile prior the transmission, and in [7]-[12] a random EH model is considered. In [3], an EH system is considered assuming that the channel state information (CSI) available at the transmitter (CSI-T) and the receiver (CSI-R) is perfect. The offline power policy that maximizes the achievable rate is *Water-Filling* in time. A battery with energy leakage is considered for the problem of the offline power scheduling for EH systems in [4]. The transmission completion time minimization problem is solved in [5] and [6]. It is shown in [5] that the solution is the same as the solution for the throughput maximization problem. In [6], the authors solve the problem taking into account both the arrivals of the harvested energy and the availability of the data over time. Considering the throughput maximization problem, the optimal transmission in the case where the EH model is random was studied in [7], using a first-order Markov process and dynamic programming. The challenge in EH systems is how to allocate the energy resources. Thus, increasing the lifetime of the energy storage device has been proposed in [8] where the authors proposed a dynamic power management policy to stabilize data queues. In order to prolong lifetime of wireless network, optimal transmission policies taking into account both transmission and processing energy costs are studied in [9]. Reference [10] suggested a sub-optimal power policies for perfect and imperfect knowledge of the energy. In [11], a Markov process was proposed to solve the throughput maximization problem with 1-bit channel feedback. The authors in [12] proposed a data-driven stochastic EH model to solve the throughput maximization problem.

B. Scope and Contribution

The idea of EH for wireless communications appears promising or even essential for establishing the 5G network. Convinced by this technology and the promise of helping our community make us more motivated to explore the performance of systems powered by energy harvested transmitters in wireless communications. As known in practical wireless communication systems, the fading channel changes randomly. Naturally, the fading level is not perfectly known at TX due to the noise in the feedback link and other limitations. The design of the optimal power allocation during transmission for different degree of CSI-T is important to yields better performance of the communication system. Also, deployment of practical EH communication systems does not seem feasible without assessing the loss incurred by imperfect CSI on the performance.

The framework of the present paper is close to the spirit of [7] and reports a significant extension introduced in this reference. More specifically, our work focuses on practical wireless systems where the CSI-T is imperfect and studies the optimal online power. To the best of our knowledge, no results capturing this setting has been reported in the literature previously. Namely, we assume that TX has an estimated version of the actual channel gain obtained, for instance, through a minimum mean square error (MMSE) filter of a feedback link. This model of CSI-T is widely accepted by the community and has been extensively adopted, e.g, [13].

We investigate the optimal online scheduling for maximum throughput by a deadline T in a setting where we take the randomness of the amounts of energy harvested.
Our contribution in this paper is as follows:
• We provide an online optimal power policy for communication system powered by energy harvester nodes under the assumption of imperfect CSI-T.
• We provide a study of the asymptotic performance of EH systems by analyzing the asymptotic behavior of the average throughput where the energy harvested amounts are very small and when they are very high for the case of imperfect CSI-T.
• We provide extensive simulations results that give an insight about the performance of EH communication systems. We also study the impact of increasing the number of sensing instants on the throughput performance.

C. Outline of the Paper

This paper is organized as follows. Section II provides the system model. In Section III, we solve the optimal power allocation problem. In Section IV, we investigate the performance of the communication system. Selected numerical results that support our analysis are presented in Section V. Section VI concludes the paper.

D. Notations

We use $\underline{v}$ to denote a vector of length N, i.e., $\underline{v} = (v_1, v_2, ..., v_N)$. The symbol "$\approx$" designates asymptotic equality, i.e., $f(x) \overset{a}{\approx} g(x)$ if and only if $\lim\limits_{x \to a} \frac{f(x)}{g(x)} = 1$. When it is clear from the context, we omit a in $\overset{a}{\approx}$ for convenience. For a Random Variable X, $f_x(\cdot)$ denote the probability density function. We use $\mathbb{P}$ to denote the probability. The set $M_n(\mathbb{R})$ represents the set of real matrices $n \times n$. For a finite set A, "$|A|$" designates the cardinal of the set A.

E. Definitions

We define:
- The average throughput (AT): the throughput of the communication system per second. i.e.,

$$AT = \frac{1}{T} \sum_{i=1}^{N} T_i, \tag{1}$$

where T_i denotes the throughput at time slot i.
In this paper, we use these notations for AT:
• AT_{Stat}: AT when the channel is static.
• $AT_{P.CSI-T}$: AT when CSI-T is perfectly known.
• $AT_{N.CSI-T}$: AT when CSI-T is unavailable.
• $AT_{I.CSI-T}$: AT when CSI-T is imperfect.
- The average recharge rate (ARR): the average of energy harvested over the deadline T. i.e.,

$$ARR = \frac{1}{T} \mathbb{E}_{\underline{E}} \left(\sum_{i=0}^{N-1} E_i \right), \tag{2}$$

where E_i denotes the amount of energy harvested at epoch i and the averaging is over all possible values of energy arrivals.

II. System Model

System description: A point-to-point data transmission during a deadline T is considered in this work, where TX in the communication system is powered by an EH sensor as shown in Fig.1. The data buffer contains always data available for transmission. The channel between TX and the receiver (RX) is possibly a fading channel. The received signal y is given by:

$$y = hx + n, \tag{3}$$

where $h \sim \mathcal{CN}(0, \sigma^2)$ is a zero mean circularly symmetric complex variable with variance σ^2 with bandwidth W, x is the channel input, and n is a zero-mean additive white Gaussian noise with spectral density N_0, and is independent of h. The energy available in the battery determines the feasible bits that can be transmitted during each time slot (TS) of duration L. If we consider a Gaussian signaling for transmission over a complex Gaussian channel, with a cost of Lp units of energy, TX sends $\frac{L}{2}\log(1 + \gamma p)$ bits of data, where p is the power used for transmission and γ is the fading level given by $\gamma = |h|^2$. We assume that the transmission is performed in TSs. During each TS, TX encodes the bits to be sent as data symbols, where the block length of each symbol is assumed to be large enough so that we can guarantee the reliability of the decoding process. A feedback link is considered between the RX and TX, the CSI feedback is sent to TX from RX and it is required only at the beginning of TSs.

Channel model: In our work, the channel is considered to be a fading channel. It varies randomly due to the environment.

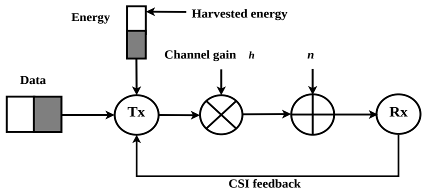

Fig. 1. An EH communication system model with CSI feedback.

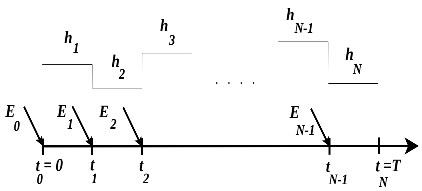

Fig. 2. Energy arrival and fading channel during a period T.

In most operating scenarios, the wireless channel varies slowly over time, so the fading level is assumed to be constant at each TS during the transmission of data. That is why, we can consider a change in the channel fading level in a discrete time $t_1, t_2, ..., t_{N-1}$ which represent the sensing instants of the energy harvester node as shown in Fig.2. The fading level during the TS i, $[t_i, t_{i+1}]$ is assumed to be equal to h_i, the length of TS i is $L_i = t_{i+1} - t_i$. We assume that during the communication perfect the instantaneous CSI-R is available. Different scenarios for the availability of the CSI-T are studied in this work. In the case where the CSI-T is perfect, we consider that changes in fading are known before the beginning of the transmission. On the other hand, if TX does not know perfectly the instantaneous CSI due to different sources of errors in the estimation process such as noise, TX estimates the actual channel as $\widehat{h}$. The channel estimation error is denoted as $\tilde{h}$ which is zero-mean and independent of $\widehat{h}$. The channel estimation model can be written as:

$$h = \sqrt{1-\alpha}\ \widehat{h} + \sqrt{\alpha}\ \tilde{h}, \tag{4}$$

where α is the error variance, $\alpha \in [0, 1]$.

Energy model: In our model, we assume that TX is equipped by a single energy harvesting sensor node. This node senses periodically in a discrete time, during the deadline T, the amount of energy available in the environment at beginning of TS $i \in \{1, 2, ..., N\}$. At t_i, E_i units of energy is harvested, and the battery is assumed to be empty initially. The transmission begins at $t_0 = 0$ when the energy E_0 is scavenged by TX. The energy is used only for data transmission, the unused energy is stored in the battery which has a limited capacity equal to E_{max} units of energy. The energy consumed for sensing at each TS, sampling and compression of data is considered negligible.

Different from the assumption that states a deterministic EH process, we take into account the randomness and the uncertainty of the amounts of the energy harvested during each TS. Since the sensing is considered periodic, the length of TSs is assumed to be constant, i.e., $L_i = L = \frac{T}{N}$. In order to capture applications based on EH networks where the TX has not a deterministic knowledge as suggested previously, we design a more sophisticated online model where we develop a Markov process that takes into consideration the unpredictable amount of the energy harvested. We assume that the energy arrival is modeled by a first-order stationary Markov chain. Therefore, TX disposes a statistical knowledge of energy arrival amounts. This statistical knowledge can be determined using measurements in practice.

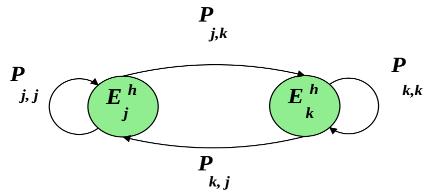

Fig. 3. Two states from the Markov chain.

The EH process is modeled as Markov chain containing N_s states which are defined prior the transmission and the possible transitions that may occur depending on their probabilities. Each state of the model depends mainly on the environment, it describes the amount of the energy harvested. The random variable E_i takes values in a finite set of states $\mathcal{E} = \{E_1^h = 0, E_2^h, ..., E_{N_s}^h\}$. In Fig.3, we show a model for two states chosen randomly from the N_s states. In fact, the states contains the zero state, i.e., $E_1^h = 0$ which corresponds to the case where no energy available in environment. The transition probabilities between states are denoted as follows:

$$\mathbb{P}(E_j^h \rightarrow E_k^h) = P_{j,k}, \forall j, k \in [\![1, N_s]\!]. \tag{5}$$

The matrix of the transition probabilities is denoted $\mathbf{M} \in M_{N_s}(\mathbb{R})$, defined by $M_{jk} = P_{j,k}$. the steady state probability in this case is given by $\pi^* = [\mathbb{P}_{E_1^h} ... \mathbb{P}_{E_{N_s}^h}]$. Since the EH process is modeled as first-order stationary Markov chain, The transition probability of this random variable is defined as follows:

$$\mathbb{P}(E_i | E_0, E_1, ..., E_{i-1}) = \mathbb{P}(E_i | E_{i-1}), \forall i \in [\![1, N-1]\!]. \tag{6}$$

One can simplify the state transition probability as:

$$\mathbb{P}(E_i | E_0, E_1, ..., E_{i-1}) = \prod_{t=1}^{i} \mathbb{P}(E_t | E_{t-1}). \tag{7}$$

Based on the statistical knowledge provided as a look-up table to TX and the amount of energy harvested in the previous TS, TX has to schedule his coding and transmission schemes to allocate optimally the energy across time in order to boost the performance.

Battery Dynamics: During the epoch i of length L, TX sends the data symbol i amplified by the power p_i. Then, TX consumes $L \times p_i$ units of energy during this epoch from the energy stored in the battery in this epoch B_i. Just after this time slot, the energy harvester scavenge an amount of energy of E_i. Hence, in the epoch $i+1$ the stored energy in the battery is updated as follows:

$$B_{i+1} = min\{B_i - Lp_i + E_i, E_{max}\}, \forall i \in [\![1, N-1]\!] \tag{8}$$

$$B_N = B_N - Lp_N. \tag{9}$$

From (8), one can conclude that the energy stored in the battery follows a first-order Markov process that depends on the immediate past energy harvested and the transmitted power. The initial amount of the energy saved in the battery B_1 is equal to the amount of energy harvested E_0 by the node that engenders the beginning of the transmission of data, i.e., $B_1 = min\{E_0, E_{max}\}$.

Energy Harvesting constraints: In order to obtain the power policy, we suppose that the consumption of the energy harvested is constrained by the energy profile. Therefore, we assume the constraints existing in energy harvesting systems [2]. Accordingly, we consider the causality constraint which means that TX can not use, at the current time, energy that has not arrived yet. Following recent works [2] and [11], we assume that the transmitted power must be kept constant in each epoch. We denote the power consumed in each epoch i by p_i. So, the causality constraint can be expressed as:

$$\sum_{j=1}^{i} Lp_j \leq \sum_{j=0}^{i-1} E_j, \forall i \in [\![1, N]\!]. \tag{10}$$

Also, one can understand this constraint as if TX can transmit only energy which is already available in the battery, i.e.,

$$0 \leq L \times p_i \leq B_i, \quad \forall i \in [\![1, N]\!]. \tag{11}$$

Because the energy arrives randomly at different time points, it is crucial to ensure that the energy level in the battery never exceeds E_{max}. This outlines the energy storage constraint on the power policy which can be expressed as:

$$\sum_{j=0}^{i} E_j - \sum_{j=1}^{i} Lp_j \leq E_{max}, \forall i \in [\![1, N-1]\!]. \tag{12}$$

Also, this constraint can be reflected by (8) in the battery dynamics, which states that the energy stored in the battery during TS i must be less than E_{max}. These constraints design a limited region of possible power policies that are feasible.

Using the Markov chain that captures the EH dynamics (6) and the first-order Markov process that captures the battery dynamics (8), we can design a new first-order Markov process where the states of this process are defined as the joint state between the EH's state and the battery's state, i.e., the state at the initial TS: $S_1 \triangleq (B_1)$ which is assumed to be known to TX, the states during TSs are defined by: $\forall j \in [\![2, N]\!]$,$S_j \triangleq (E_{j-1}, B_j)$ and a last state $S_{N+1} \triangleq (B_{N+1})$ which describes the state of battery by the end of transmission. Hence, one can deduce from (6) and (8) that:

$$\mathbb{P}(S_i | S_1, S_2, ..., S_{i-1}) = \mathbb{P}(S_i | S_{i-1}), \forall i \in [\![2, N+1]\!]. \tag{13}$$

Consequently, the state transition probability between these states can be rewritten as follows:

$$\mathbb{P}(S_i | S_1, S_2, ..., S_{i-1}) = \prod_{k=2}^{i} \mathbb{P}(S_k | S_{k-1}), \forall i \in [\![2, N+1]\!]. \tag{14}$$

Therefore, our objective is to formulate the throughput maximization problem given a deadline T subject to EH constraints for different assumptions on the availability of the CSI-T.

III. Optimal Power Policy

In this section, we determine the optimal power policy that maximizes the throughput of the EH communication system.

A. Problem Formulation

Our throughput maximization problem can be classified as a discrete-time Markov Decision Process (MDP). In fact, at the initiation of the communication TX has the information about the initial energy harvested. Then, at the beginning of each TS, an EH TX decides to transmit with power p_i at the beginning of the TS i, based on its observations on the system state taking into consideration the matrix of the transition probabilities. Our aim is to find an optimal and feasible power policy that maximizes the expected reward function, i.e., the cumulative data rate during N TSs, i.e.,

$$(P): \max_{\{p_i\}_{i=1}^{N}} \mathbb{E}_{S_2^N} \left\{ \mathbb{E}_{\underline{h}} \left[\sum_{i=1}^{N} C(h_i, p_i) \right] \mid \mathbf{M}, S_1 \right\} \tag{15}$$

$$\text{s.t.} \begin{cases} 0 \leq Lp_i \leq B_i, \forall i \in [\![1, N]\!] \\ B_{i+1} = min\{B_i - Lp_i + E_i, E_{max}\}, \forall i \in [\![1, N-1]\!] \\ B_{N+1} = B_N - Lp_N, \end{cases}$$

where $C(h_i, p_i)$ denotes the information capacity between TX and RX. $\mathbb{E}_{S_2^N}$ designates the statistical expectation over all possible states during TSs $i = 2, ..., N$. $\mathbb{E}_{\underline{h}}$ designates the statistical expectation over the random variable h. For the rest of our paper, we will assume that TX transmits symbols over a Gaussian channel. Hence, the achievable rate during TS i in this case is:

$$C(h_i, p_i) = \frac{L}{2} \log(1 + |h_i|^2 p_i) = \frac{L}{2} \log(1 + \gamma_i p_i). \tag{16}$$

Generally, the optimization problem in (15) cannot be solved independently for each time slot due to the dependence of constraints along TSs. For instance, the energy consumed currently affects the energy stored in the battery in the next TS, and therefore affects the future power allocation. Consequently, such sequential optimization problem with random EH amounts, we can solve it optimally using finite-horizon dynamic programming (DP).

B. Online Power Policies with Fading Channel

In this section, we solve the problem (15) by using finite-horizon DP. In fact, the optimal power allocation is determined by backward induction method [14]. For instance, $\{p_1^*, p_2^*, ..., p_N^*\}$ is calculated in the time reversal order. The reward function can be understood as the maximal sum of the data rate in the current TS, and taking into account the matrix $\mathbf{M}$ we maximize the expected cumulative data rate in the future TSs resulted from the current state and the current transmitted power. To the best of our knowledge, no results capturing the setting of imperfect CSI-T has been reported in the literature previously. For this case, we consider the channel estimation model given by (4). Considering a Gaussian channel, we define the online point-to-point throughput maximization problem subject to constraints (11) and (8) of an energy harvesting system as:

$$(P_1): \max_{\{p_i\}_{i=1}^N} \mathbb{E}_{S_2^N}\left\{\mathbb{E}_{\underline{\gamma}|\underline{\hat{\gamma}}}\left[\sum_{i=1}^N \frac{L}{2}\log(1+\gamma p_i)\right] \mid \mathbf{M}, S_1\right\} \quad (17)$$

$$\text{s.t.} \begin{cases} 0 \le Lp_i \le B_i, \forall i \in [\![1, N]\!] \\ B_{i+1} = min\{B_i - Lp_i + E_i, E_{max}\} \\ B_{N+1} = B_N - Lp_N. \end{cases}$$

The reward functions are calculated recursively as follows:

1) The last TS, $i = N$**:**

$$R_N(\hat{\gamma}_N, S_N, \alpha) = \max_{p_N} \mathbb{E}_{\gamma|\hat{\gamma}_N}\left[\frac{L}{2}\log(1+\gamma p_N) \mid \mathbf{M}, S_1\right] \quad (18)$$
$$\text{s.t. } 0 \le Lp_N \le B_N.$$

Proposition 1. The optimal last state S_{N+1}^* of the first-order Markov process is $S_{N+1}^* \triangleq (B_{N+1} = 0)$.

The Proposition 1 states that TX transmits the last symbol with full energy available in the battery, this result is quite consistent with recent works where the causality constraint (12) is assumed to be satisfied with equality at the last time slot, e.g., [3]. Therefore, the reward function in the last time slot can be evaluated using $f_{\gamma|\hat{\gamma}}(\cdot)$ as follows:

$$R_N(\widehat{\gamma}_N, S_N, \alpha) = \frac{L}{2}\mathbb{E}_{\gamma|\widehat{\gamma}_N}\left[\log(1+\gamma\frac{B_N}{L})\right]. \quad (19)$$

As a consequence, by the end of the communication, we have:

$$\sum_{i=0}^{N-1} E_i = \sum_{i=1}^{N} Lp_i^*. \quad (20)$$

2) The TSs, $i = N-1, N-2, ..., 1$**:** During the TS i, the optimal power is determined by maximizing the reward function corresponding to this TS which is expressed as follows:

$$R_i(\widehat{\gamma}_i, S_i, \alpha) = \max_{p_i}\left\{\mathbb{E}_{\gamma|\widehat{\gamma}_i}\left[\frac{L}{2}\log(1+\gamma p_i)\right] + \overline{R}_{i+1}(\widehat{\gamma}_{i+1}, S_{i+1}, \alpha)\right\} \quad (21)$$
$$\text{s.t. } 0 \le Lp_i \le B_i,$$

where $\overline{R}_{i+1}(\widehat{\gamma}_{i+1}, S_{i+1}, \alpha)$ designates the expected values of the future reward functions and it is defined as follows:

$$\overline{R}_{i+1}(\widehat{\gamma}_{i+1}, S_{i+1}, \alpha) = \mathbb{E}_{\mathcal{S}^i}\left\{\mathbb{E}_{\gamma|\widehat{\gamma}_{i+1}}\left[R_{i+1}(\widehat{\gamma}_{i+1}, S_{i+1}, \alpha)\right] |\mathbf{M}, S_i\right\}, \quad (22)$$

where $\mathbb{E}_{\mathcal{S}^i}$ designates the statistical expectation over the set of possible states in the TS $i+1$ given the state S_i and the transition probabilities. The reward functions $R_i(\widehat{\gamma}_i, S_i, \alpha)$ are not available as closed form expressions. A numerical computation will provide us an optimal power policy.

Theorem 1. *Given a channel fading level estimation at the different TSs denoted by the vector* $\hat{\gamma} = (\hat{\gamma}_1, \hat{\gamma}_2, ..., \hat{\gamma}_N)$*, the initial state* $S_1 \triangleq (B_1) \triangleq (E_0)$ *and the matrix of transition probabilities* $\mathbf{M}$ *related to Markov chain that models the EH process. The optimal power allocation for TSs* $i = 1, 2, ..., N$ *is given by the initial reward function* $R_1(\underline{\widehat{\gamma}}, S_1, \alpha)$*.*

Proof. The proof follows immediately by applying equations of Bellman [15] and the equations that resume the dynamic of the Markov process ((8) and (6)). □

1) Some properties of optimal power policy:

Proposition 2. Under settings and reward functions defined previously, one can deduce that $R_i(\widehat{\gamma}_i, S_i, \alpha)$ and $\overline{R}_i(\widehat{\gamma}_i, S_i, \alpha)$ are continuous, non-decreasing and concave in B_i.

Proof. We note that studying the continuity, the non-decreasing and the convexity properties of the reward functions in the belief state is quite well known in the MDP literature [16] [17], thus the proof is omitted. □

The Proposition 2 states that the optimal cumulative data rate is concave and increasing in the initial state of the battery. This result is quite intuitive, the higher amount of energy harvested at the beginning of the transmission, the higher the maximum achievable expected data rate.

Remark 1. One can see easily that the optimal reward function $R_1(\widehat{\gamma}_1, S_1, \alpha)$ is non-increasing in α. This can be deduced immediately from the definition of the parameter α which measures the quality of the CSI-T.

We note that our framework can be used to depict some specific scenarios (the perfect CSI-T and the no CSI-T cases). Hence, our work is a generalization of maximizing the throughput considering the fading channel case.

2) Special cases:

a) Online optimal power policy with perfect CSI-T: Using the framework presented previously, one can determine the optimal power allocation when TX has a perfect knowledge of the channel during TSs. In order to capture this case, one can set $\alpha = 0$ and $f_{\gamma|\hat{\gamma}} = \delta(\gamma - \hat{\gamma})$. Hence, $\{p_1^*, p_2^*, ..., p_N^*\}$ is calculated recursively using the same procedure as the imperfect CSI-T case.

b) Online optimal power policy without CSI-T: In some situations, having an estimate of the channel is challenging due to the huge amount of noise in the environment. Then, the optimal power allocation in such situations can determined by our framework by setting $\alpha = 1$ and $f_{\gamma|\hat{\gamma}} = f_\gamma$. Hence, $\{p_1^*, p_2^*, ..., p_N^*\}$ is calculated recursively using the same procedure as the imperfect CSI-T case.

c) Online optimal power policy with static channel: This work can be extended also to retrieve the optimal reward function when a channel is considered to be constant equal to $\overline{\gamma}$ during TSs.

Theorem 2. *Given a random EH process and the matrix of transition probabilities* $\mathbf{M}$*, the optimal policy when the CSI is unavailable at TX is the same as the power policy in the case when the channel is static during communication.*

Proof. See Appendix A. □

The Theorem 2 states that for the case where the channel is assumed to be constant along time and the case where the CSI-T is unavailable, the optimal power policy is the same. One can use this result to determine the optimal reward function when the channel is constant $\overline{\gamma}$. For instance, performing the optimization problem under the setting $\alpha = 1$ will provide a look up table where optimal powers are stored. Then, it is easy to evaluate the optimal reward function by plugging the optimal power policy in the objective function (37) and setting $\alpha = 0$ and $\forall i \in [\![1, N]\!], \quad \gamma_i = \overline{\gamma}$.

IV. Performance Analysis

In this section, average throughput performance is studied when the CSI-T is partially available. Given an error variance α of the channel estimation, the average throughput of the communication system can be written as follows: $\forall\ \alpha \in [0, 1]$,

$$\begin{aligned} AT_{I.CSI-T}(\alpha) &= \frac{1}{T}\mathbb{E}_{S_1}\left\{\mathbb{E}_{\hat{\underline{\gamma}}}\left(R_1(\hat{\underline{\gamma}}, S_1, \alpha)\right)\right\} \\ &= \mathbb{E}_{S_1}\left\{\mathbb{E}_{S_2^N}\left\{\mathbb{E}_{\hat{\underline{\gamma}}}\left(\sum_{i=1}^{N}\frac{L}{2T}\mathbb{E}_{\gamma|\hat{\gamma}_i}\left[\log(1+\gamma p_i^*(\hat{\underline{\gamma}}))\right]\right) \mid \mathbf{M}, S_1\right\}\right\} \\ &= \mathbb{E}_{S_1^N}\left\{\mathbb{E}_{\hat{\underline{\gamma}}}\left(\sum_{i=1}^{N}\frac{L}{2T}\mathbb{E}_{\gamma|\hat{\gamma}_i}\left[\log(1+\gamma p_i^*(\hat{\underline{\gamma}}))\right]\right) \mid \mathbf{M}\right\}, \end{aligned} \tag{23}$$

where $\mathbb{E}_{S_1^N}$ designates the statistical expectation over all possible states during TSs $i = 1, ..., N$.

Based on the developed optimal power policy, we present, in the following, our asymptotic analysis that captures two extremes cases, the high ARR regime ($ARR \to +\infty$) and the low ARR regime ($ARR \to 0$), respectively.

A. Low ARR Regime

In this subsection, we evaluate the performance of the communication system when the energy harvested is scarce. We focus of utmost interest in such regime to capture applications where the communication system is implanted in an environment where the amount of energy harvested in each state is scarce.

Proposition 3. Given that all energies are very low. Given that the channel between TX and RX is a fading channel where the CSI-T is unavailable and a static channel equal to $\overline{\gamma}$, respectively. The AT of the system grows linearly with ARR as ARR decreases to zero, respectively, i.e.,

$$AT_{N.CSI-T} \approx \frac{\mathbb{E}_{\tilde{\gamma}}(\tilde{\gamma})}{2}ARR \tag{24}$$

$$AT_{Stat} \approx \frac{\overline{\gamma}}{2}ARR. \tag{25}$$

Proof. In the low ARR regime, we assume that all powers are very low. In the case where the CSI-T is unavailable, by setting $\alpha = 1$, we have:

$$AT_{N.CSI-T} = \mathbb{E}_{S_1^N}\left\{\mathbb{E}_{\tilde{\gamma}}\left[\sum_{i=1}^{N}\frac{L}{2T}\log(1+\tilde{\gamma}p_i^*)\right] \mid \mathbf{M}\right\} \tag{26}$$

$$\approx \frac{1}{2}\mathbb{E}_{S_1^N}\left\{\mathbb{E}_{\tilde{\gamma}}\left[\sum_{i=1}^{N}\frac{L}{T}p_i^*\tilde{\gamma}\right] \mid \mathbf{M}\right\} \tag{27}$$

$$= \frac{1}{2}\mathbb{E}_{S_1^N}\left\{\sum_{i=1}^{N}\frac{L}{T}p_i^* \mid \mathbf{M}\right\}\mathbb{E}_{\tilde{\gamma}}(\tilde{\gamma}). \tag{28}$$

Using (20), then we have:

$$AT_{N.CSI-T} = \frac{\mathbb{E}_{\tilde{\gamma}}(\tilde{\gamma})}{2}\mathbb{E}_{S_1^N}\left\{\sum_{i=0}^{N-1}\frac{E_i}{T} \mid \mathbf{M}\right\} \tag{29}$$

$$= \frac{\mathbb{E}_{\tilde{\gamma}}(\tilde{\gamma})}{2T}\sum_{i=0}^{N-1}\sum_{j=1}^{N_s}\mathbb{P}_{E_j^h}E_j^h \tag{30}$$

$$= \frac{\mathbb{E}_{\tilde{\gamma}}(\tilde{\gamma})}{2T}\mathbb{E}_{\underline{E}}\left\{\sum_{i=0}^{N-1}E_i\right\} = \frac{\mathbb{E}_{\tilde{\gamma}}(\tilde{\gamma})}{2}\ ARR. \tag{31}$$

The expression of the AT_{Stat} can be proved using similar lines of the proof for $AT_{N.CSI-T}$. □

The Proposition 3 asserts that with no CSI-T, no gain is provided by fading in terms of AT given that the expected value of the fading level is equal to $\overline{\gamma}$.

Theorem 3. *Given an error variance α, the AT in the low ARR regime is a combination of the AT when the fading level of the channel is known perfectly and the AT when it is unavailable at TX, i.e., $\forall\alpha \in [0, 1]$,*

$$AT_{I.CSI-T}(\alpha) \approx (1-\alpha)\ AT_{P.CSI-T} + \alpha\ AT_{N.CSI-T}. \tag{32}$$

Proof. See Appendix B. □

B. High ARR Regime

In this subsection, we evaluate the performance of the communication system when the energy harvested is abundant. This assumption captures applications where the communication system is running in an environment where the amount of energy harvested in each state is very high. In such scenario, the state $E_1^h = 0$ is omitted because the node can harvest a very high amount of energy at each sensing. Then, the AT is investigated in the high ARR regime.

Proposition 4. Provided that all powers are very high in the high ARR regime, then the AT in the when the CSI-T is imperfect increases as follows:

$$AT_{I.CSI-T}(\alpha) \approx AT_{Stat} + \frac{\mathbb{E}\left[\log(\gamma)\right]}{2}, \qquad \forall\alpha \in [0, 1]. \tag{33}$$

Proof. See Appendix C □

The interpretation of this result is that, at high ARR, the AT when the CSI-T is imperfect increases similarly as the AT in the case where the channel is static over all epochs, irrespective of the value of α. According to the previous result, the gap between $AT_{I.CSI-T}$ and AT_{Stat} is constant at high ARR, and the loss depends on the property of the fading distribution. Consequently, knowing the channel state when the ARR is high has no gain. In fact, TX can achieve the same performance in terms of data transmitted regardless of its channel knowledge. Hence, TX can utilize during the communication the same power profile as the worst case ($\alpha = 1$) to achieve a similar performance as the best case ($\alpha = 0$).

C. Large Number of Time Slots N

In this subsection, we study the impact of the large number of TSs, during the transmission, on the AT. The sensor node performs the sensing periodically with period equal to $\frac{T}{N}$sec. One can remark that when N increases, the throughput increases as well, because more energy is available for transmission of data that can enhance the performance of the system. However, the AT can not keep increasing while N increases, because it is upper bounded as stated in the following proposition.

Proposition 5. Beyond a very large number N, the average throughput will saturate to a finite quantity, i.e.,

$$\lim_{N\to+\infty} AT_{I.CSI-T}(\alpha) = l < +\infty, \tag{34}$$

where $l < \mathbb{E}_{\underline{\hat{\gamma}}}\left(\mathbb{E}_{\gamma|\underline{\hat{\gamma}}}\left[\log(1+\gamma P_{Avg}\right]\right)$ and $P_{Avg} = ARR$.

Proof. Using Jensen inequality, we have:

$$\mathbb{E}_{\gamma|\underline{\hat{\gamma}}}\left[\sum_{i=1}^{N}\frac{L}{2T}\log(1+\gamma p_i^*(\underline{\hat{\gamma}}))\right] \leq \frac{1}{2}\mathbb{E}_{\gamma|\underline{\hat{\gamma}}}\left[\log(1+\gamma\sum_{i=1}^{N}\frac{Lp_i^*(\underline{\hat{\gamma}})}{T})\right].$$

$$\sum_{i=1}^{N}\frac{Lp_i^*(\underline{\hat{\gamma}})}{T} = \frac{1}{T}\sum_{i=0}^{N-1}E_i$$

On the other hand,

$$\sum_{i=1}^{N}\frac{Lp_i^*(\underline{\hat{\gamma}})}{T} = \frac{1}{N}\sum_{i=1}^{N}p_i^*(\underline{\hat{\gamma}}) \underset{N\to+\infty}{\longrightarrow} P_{Avg}$$

Thus,

$$\mathbb{E}_{\gamma|\underline{\hat{\gamma}}}\left[\sum_{i=1}^{N}\frac{L}{2T}\log(1+\gamma p_i^*(\underline{\hat{\gamma}}))\right] \leq \mathbb{E}_{\underline{\hat{\gamma}}}\left(\mathbb{E}_{\gamma|\underline{\hat{\gamma}}}\left[\log(1+\gamma P_{Avg}\right]\right)$$

The upper bound in last equation is independent of the states. Hence, it is an upper bound for the AT. Also, one can see the AT as a series of non-negative terms. Since the sequence of the partial sum is bounded, we deduce (34) immediately. □

Intuitively, the upper bound is asymptotically achievable for large N. For instance, large number N corresponds to a higher energy-sensing frequency. Since the sensing is performed in the same space of states and the duration between two consecutive sensing is asymptotically equal to 0. Consequently, the EH sensing can be considered in this case as a continuous process in time. Hence, the communication system can always harvest energy during the deadline T, as if it has the ability to scavenge "infinitely" the energy.

V. Simulation Results

In this section, we present some selected numerical results that illustrate our theoretical analysis. Indeed, we consider a point to point communication system in order to compare the average throughput of this system, for different scenarios of CSI-T availability, during the transmission of data under energy harvesting constraints.

A. Model for Simulation Results

In order to obtain the numerical results, the deadline of the transmission is fixed $T = 10$ sec, also we consider a band-limited additive white Gaussian noise channel, with bandwidth W chosen $W = 1$ MHZ for simulations and the noise power spectral density is $N_0 = 10^{-19}$ W/Hz, with $h, \hat{h}$ and $\tilde{h} \sim \mathcal{CN}(0,1)$. Consequently, γ, $\hat{\gamma}$ and $\tilde{\gamma}$ have an exponential distributions with unit mean. All the simulations are performed for 5×10^4 realizations of channels. As mentioned, in our considered model, the data rate sent to RX at TS i is calculated as $C(h_i, p_i) = \frac{W}{2}\log_2(1+\gamma_i p_i)$ bits/sec. We assume that the energy takes values in a finite set, i.e., $N_s = |\mathcal{E}| = 3$, for simulations we fix $\mathcal{E} = \{E_1^h = 0, E_2^h = \beta, E_3^h = \eta\}$, such that $0 < \beta < \eta$. For EH process, we assume that the matrix of the transition probabilities $\mathbf{M}$ is defined as follows:

$$\mathbf{M} = \frac{1}{3}\begin{pmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{pmatrix}. \tag{35}$$

It is clear that the steady state probability in this case is given by $\pi^* = [\mathbb{P}_{E_1^h}\mathbb{P}_{E_2^h}\mathbb{P}_{E_3^h}] = \frac{1}{3}[1\ 1\ 1]$. Thus, in this case, the average energy that can be harvested at instant t_i is defined as follows:

$$\overline{E} = \mathbb{E}(E_i) = \sum_{j=1}^{N_s=3}\mathbb{P}_{E_j^h}E_j^h = \frac{\beta+\eta}{3}. \tag{36}$$

The maximum energy that can be supported by the battery is $E_{max} = 100$ J.

B. Characteristics of Optimal Policy

The optimal power policy is determined by induction method. Fixing an error estimation variance α, the powers $p_1^*, p_2^*, ..., p_N^*$ are calculated recursively in the time reversal order. Starting by the evaluation of the reward function at the last TS N, using the closed form expression given in (19). Then, we calculate $\overline{R}_N(\widehat{\gamma}_N, S_N, \alpha)$ using (22), for different battery capacity B_N discretized in an adaptive step size depending on the regime where the system is running (low, medium and high ARR regime). We set the step size $\Delta B = 0,02$ J to plot figures 4, 5 and 7 and it is chosen $\Delta B = 2 \times 10^{-7}$J to plot Fig.6. Also, $\overline{R}_N(\widehat{\gamma}_N, S_N, \alpha)$ is averaged over 5×10^4 channel realizations. After, we calculate $R_{N-1}(\widehat{\gamma}_{N-1}, S_{N-1}, \alpha)$ solving (21) using a one line search method. The value of $R_{N-1}(\widehat{\gamma}_{N-1}, S_{N-1}, \alpha)$ is stored as a look-up table to be used for the next TSs. Then, the same procedure is applied for TSs $N-2, ..., 1$. At TS 1, TX determines the optimal reward function by evaluating $R_1(\underline{\hat{\gamma}}, S_1, \alpha)$. To evaluate the optimal AT, the reward function is averaged over 5×10^4 independent realizations of $\underline{\hat{\gamma}}$ and over all possible states for the initial state S_1.

We start by examining the AT with different values of the parameter α in order to depict the performance of the communication system for different scenarios of CSI-T availability. In Fig.4, we can see that the AT is a non-increasing function in α as stated in our analytical claims. Also, we can observe that

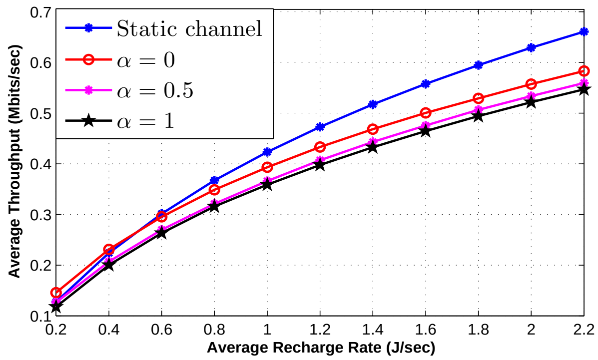

Fig. 4. Performances of the online policies for various energy arrival rates under exponential fading channel, $\beta = 0.2$ J and $N = 5$.

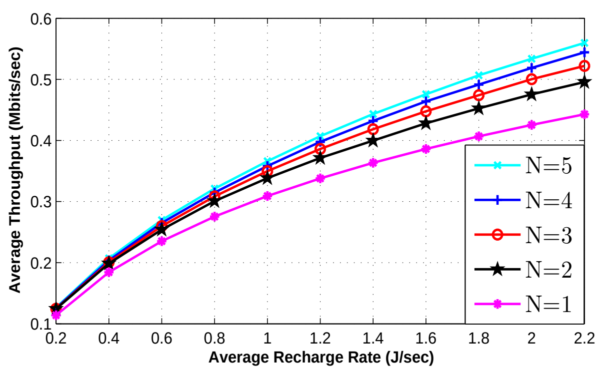

Fig. 5. Performances of the online policies for various number of TSs $\beta = 0.2$ J and $\alpha = 0.5$.

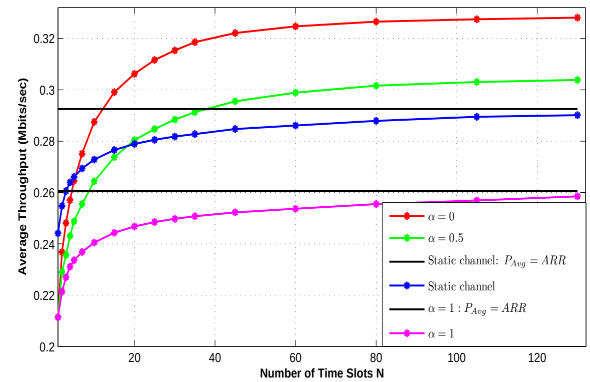

Fig. 6. Performances of the online policies for large N, $ARR = 0.1$ J.

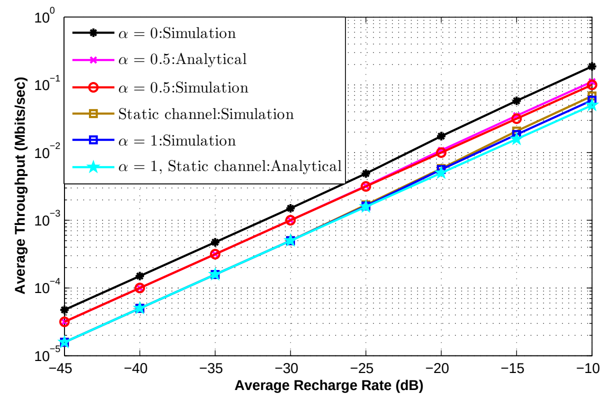

Fig. 7. Performances of the online policies at low ARR regime, $\beta = 3.16 \times 10^{-5}$ J and $N = 20$.

the EH system with a static channel equal to $\overline{\gamma} = 1$, performs better than the one experiencing fading. However, in the low ARR regime, we remark that the fading channel has a better performance (red curve). This points out the benefit provided by the fading to the EH system at low ARR. Moreover, we notice that even for a fading case where $\alpha = 0,5$ the AT does not exceed the static case. This highlights the gain provided by the available CSI-T. Indeed, this motivates us to investigate of utmost interest this gain at low ARR. Note that the performance depends on the states of the Markov chain associated to the EH process. In fact, having the same ARR does not mean having the same performance. In Fig.4, we set $\beta = 0,2$ and the third state is fixed by the ARR. In Fig.5, we investigate the effect of varying the number of TSs N on the AT. As we remark, the EH communication system performs better as N increases. This is consistent with what we claim in previous parts of this paper. Simulations in this figure are performed for a fading channel with error variance of the channel estimation $\alpha = 0,5$. As discussed previously, simulations in Fig.4 stir us to boost our analysis on the low ARR regime presented in Fig.6. Such analysis is reasonable and very useful for EH applications that are running at low ARR. We observe in Fig.6 that results found are quite consistent with our theoretical claims. In fact, the EH system take advantage from the partial knowledge of the channel to enhance its performance. We can see that when the channel experiences fading without CSI-T, the communication system has almost the same performance as the case where the channel is static with $\overline{\gamma} = 1$, (the brown and the blue curves). Using result derived in (24) to plot the cyan curve and the simulation results, thus we can conclude that our theoretical claims are confirmed. Also, we plot the AT using expression (32) (magenta curve) for $\alpha = 0,5$, we remark that this curve is close to the simulated AT for this settings. Next, we investigate the performance of the EH for large sensing instants. We observe in Fig.7 that the EH system performs better when the channel experiences fading. This is reasonable because, the ARR assumed for this simulation is considered low. Furthermore, we remark that beyond a certain value of N the AT saturates. For the two specific cases (static, no CSI-T), the upper bound stated in previous section is achievable. The gap between the $AT_{I.CSI-T}$ and AT_{Stat} is constant for very large N.

VI. Conclusion

In this paper, we have presented a comprehensive analysis on the performance of EH systems. We have considered a point-to-point data transmission with CSI feedback where TX equipped by an EH node and the CSI-R is perfect. We have proposed an optimal power allocation when the CSI-T is imperfect. For instance, using a first-order Markov process and DP framework, optimal powers are determined via a backward induction method. Furthermore, an analysis of the AT has been provided when ARR is either very low or very high. Afterwards, we have focused on the performance of the communication system when the number of TSs is very large. The simulations results have confirmed results found by our analytical analysis.

Appendix A
Proof of Theorem 2

Let us denote $(p_i^*)_i$ the optimal powers for the throughput maximization problem in the case where the channel is static, i.e., the powers maximize the following objective function:

$$\max_{\{p_i\}_{i=1}^N} \mathbb{E}_{S_2^N}\left[\sum_{i=1}^N \frac{L}{2}\log(1+\overline{\gamma}p_i) \mid \mathbf{M}, S_1\right]. \qquad (37)$$

The problem in the no CSI-T setting is:

$$\max_{\{p_i\}_{i=1}^N} \mathbb{E}_{S_2^N}\left\{\mathbb{E}_{\gamma}\left[\sum_{i=1}^N \frac{L}{2}\log(1+\gamma p_i)\right] \mid \mathbf{M}, S_1\right\}. \qquad (38)$$

Since $(p_i^*)_i$ is the optimal power policy when the channel is static, and it is clear form the objective function that the optimal powers do not depend on the channel gain $\overline{\gamma}$. The optimal solution depends only on the energies harvested along time. Thus, $(p_i^*)_i$ does not depend on the static channel gain, then we have :

$$\sum_{i=1}^N \frac{L}{2}\log(1+\gamma p_i) \leq \sum_{i=1}^N \frac{L}{2}\log(1+\gamma p_i^*),\ \forall\gamma \qquad (39)$$

$$\mathbb{E}_{S_2^N}\left[\sum_{i=1}^N \frac{L}{2}\log(1+\gamma p_i) \mid \mathbf{M}, S_1\right] \leq \mathbb{E}_{S_2^N}\left[\sum_{i=1}^N \frac{L}{2}\log(1+\gamma p_i^*) \mid \mathbf{M}, S_1\right] \qquad (40)$$

$$\max_{\{p_i\}_{i=1}^N} \mathbb{E}_{\gamma}\left\{\mathbb{E}_{S_2^N}\left[\sum_{i=1}^N \frac{L}{2}\log(1+\gamma p_i) \mid \mathbf{M}, S_1\right]\right\}$$
$$\leq \mathbb{E}_{\gamma}\left\{\mathbb{E}_{S_2^N}\left[\sum_{i=1}^N \frac{L}{2}\log(1+\gamma p_i^*) \mid \mathbf{M}, S_1\right]\right\}. \qquad (41)$$

So, $\mathbb{E}_{\gamma}\left\{\mathbb{E}_{S_2^N}\left[\sum_{i=1}^N \frac{L}{2}\log(1+\gamma p_i^*) \mid \mathbf{M}, S_1\right]\right\}$ is an upper bound for the problem (38) that can be achieved by the power policy $(p_i^*)_i$, also this policy is feasible because it satisfies the EH constraints. Consequently, the optimal power policy is $(p_i^*)_i$.

Appendix B
Proof of Theorem 3

In the low ARR regime, the powers are very low according to the causality constraint. We denote $\underline{\hat{\gamma}} = (\hat{\gamma}_1, \hat{\gamma}_2, ..., \hat{\gamma}_N)$. So we have :

$$AT_{I.CSI}(\alpha) = \mathbb{E}_{S_1^N}\left\{\mathbb{E}_{\underline{\hat{\gamma}}}\left(\sum_{i=1}^N \frac{L}{2T}\mathbb{E}_{\gamma|\hat{\gamma}_i}\left\{\log(1+\gamma p_i^*(\underline{\hat{\gamma}}))\right\}\right) \mid \mathbf{M}\right\}$$
$$\approx \mathbb{E}_{S_1^N}\left\{\mathbb{E}_{\underline{\hat{\gamma}}}\left(\sum_{i=1}^N \frac{L}{2T}\mathbb{E}_{\gamma|\hat{\gamma}_i}\left(\gamma p_i^*(\underline{\hat{\gamma}})\right)\right) \mid \mathbf{M}\right\} \qquad (42)$$
$$= \mathbb{E}_{\underline{\hat{\gamma}}}\left(\sum_{i=1}^N \mathbb{E}_{S_1^N}\left\{\frac{Lp_i^*(\underline{\hat{\gamma}})}{2T} \mid \mathbf{M}\right\}\mathbb{E}_{\gamma|\hat{\gamma}_i}(\gamma)\right) \qquad (43)$$
$$= \mathbb{E}_{\underline{\hat{\gamma}}}\left(\sum_{i=1}^N \mathbb{E}_{S_1^N}\left\{\frac{Lp_i^*(\underline{\hat{\gamma}})}{2T} \mid \mathbf{M}\right\}((1-\alpha)\hat{\gamma}_i + \alpha\mathbb{E}_{\tilde{\gamma}}(\tilde{\gamma}))\right) \qquad (44)$$
$$= (1-\alpha)\ \mathbb{E}_{\underline{\hat{\gamma}}}\left(\mathbb{E}_{S_1^N}\left\{\sum_{i=1}^N \frac{Lp_i^*(\underline{\hat{\gamma}})\hat{\gamma}_i}{2T} \mid \mathbf{M}\right\}\right)$$
$$+\alpha\ \ \mathbb{E}_{\underline{\hat{\gamma}}}\left(\mathbb{E}_{\tilde{\gamma}}(\tilde{\gamma})\ \mathbb{E}_{S_1^N}\left\{\sum_{i=1}^N \frac{Lp_i^*(\underline{\hat{\gamma}})}{2T} \mid \mathbf{M}\right\}\right). \qquad (45)$$

Using (20), then we can write the second term:

$$\mathbb{E}_{\underline{\hat{\gamma}}}\left(\mathbb{E}_{\tilde{\gamma}}(\tilde{\gamma})\ \mathbb{E}_{S_1^N}\left\{\sum_{i=1}^N \frac{Lp_i^*(\underline{\hat{\gamma}})}{2T} \mid \mathbf{M}\right\}\right) \qquad (46)$$
$$= \frac{\mathbb{E}_{\tilde{\gamma}}(\tilde{\gamma})}{2T}\sum_{i=0}^{N-1}\sum_{j=1}^{N_s}\mathbb{P}_{E_j^h}E_j^h \qquad (47)$$
$$= \frac{\mathbb{E}_{\tilde{\gamma}}(\tilde{\gamma})}{2T}\mathbb{E}_{\underline{E}}\left\{\sum_{i=0}^{N-1}E_i\right\} = \frac{\mathbb{E}_{\tilde{\gamma}}(\tilde{\gamma})}{2}\ ARR \approx AT_{N.CSI-T}. \qquad (48)$$

Using the approximation $\log(1+x) \underset{0}{\approx} x$, we have:

$$AT_{P.CSI-T} = AT_{I.CSI-T}(0) \qquad (49)$$
$$= \mathbb{E}_{S_1^N}\left\{\mathbb{E}_{\underline{\hat{\gamma}}}\left[\sum_{i=1}^N \frac{L}{2T}\log(1+\hat{\gamma}_i p_i^*(\underline{\hat{\gamma}}))\right] \mid \mathbf{M}\right\} \qquad (50)$$
$$\approx \mathbb{E}_{\underline{\hat{\gamma}}}\left(\mathbb{E}_{S_1^N}\left\{\sum_{i=1}^N \frac{Lp_i^*(\underline{\hat{\gamma}})\hat{\gamma}_i}{2T} \mid \mathbf{M}\right\}\right). \qquad (51)$$

Then (32) follows immediately.

Appendix C
Proof of Proposition 4

Let a fading channel with $\underline{\hat{\gamma}} = (\hat{\gamma}_1, \hat{\gamma}_2, ..., \hat{\gamma}_N)$ be an estimates of the fading level during transmission.

Let $\alpha = 1$, the TX have not information about the channel, we proved in Theorem 2 that the optimal power allocation is the same as the case where the channel is static. Then, we have:

$$AT_{N.CSI-T} = \mathbb{E}_{S_1^N}\left\{\mathbb{E}_{\tilde{\gamma}}\left(\sum_{i=1}^N \frac{L}{2}\log(1+\tilde{\gamma}p_i^*)\right) \mid \mathbf{M}\right\} \qquad (52)$$
$$\approx \frac{1}{2}\mathbb{E}_{S_1^N}\left\{\sum_{i=1}^N \frac{L}{T}\mathbb{E}_{\tilde{\gamma}}\left(\log(\tilde{\gamma}p_i^*)\right) \mid \mathbf{M}\right\} \qquad (53)$$
$$= \frac{1}{2}\mathbb{E}_{S_1^N}\left\{\sum_{i=1}^N \frac{L}{T}(\log(p_i^*) + \mathbb{E}_{\tilde{\gamma}}(\log(\tilde{\gamma}))) \mid \mathbf{M}\right\} \qquad (54)$$
$$\approx AT_{Stat} + \frac{\mathbb{E}_{\tilde{\gamma}}\left[\log(\tilde{\gamma})\right]}{2}. \qquad (55)$$

Lemma 1. Given the same initial state, the optimal power policy when the channel is perfectly known is related to the optimal power policy when the channel is static. In fact, the optimal powers satisfy the property:

$$\mathbb{E}_{\underline{\hat{\gamma}}}\left\{p_i^*(\underline{\hat{\gamma}})\right\} = p_{stat,i}^*, \tag{56}$$

where, $\underline{\hat{\gamma}}$ is a channel realization and $p_{stat,i}^*$ is the optimal power delivered during the TS i in the case where the channel is static.

Proof. Let us evaluate $\mathbb{E}_{\underline{\hat{\gamma}}}\left\{p_i^*(\underline{\hat{\gamma}})\right\}$ between two successive energy arrivals E_{i-1} and E_i. We have the average is over possible values of the channel, so $\mathbb{E}_{\underline{\hat{\gamma}}}\left\{p_i^*(\underline{\hat{\gamma}})\right\}$ depends only on the energy arrivals. Therefore, to maximize the throughput, the average energy (averaging with respect to the channel) consumed during the TS i between the two energy arrivals should be allocated optimally. Since, the average energy used does not depend on the channel, an optimal energy profile should be similar to the optimal energy profile when the CSI-T is unavailable. Using result stated in Theorem 2, the average energy consumed during this TS should be equal to the energy used when the channel is static between the two instants of energy arrivals. So, we have:

$$\mathbb{E}_{\underline{\hat{\gamma}}}\left\{Lp_i^*(\underline{\hat{\gamma}})\right\} = Lp_{stat,i}^*.$$

Hence, Lemma 1 follows immediately. □

Now, let us evaluate the AT when $\alpha = 0$:

$$AT_{P.CSI-T} = \mathbb{E}_{S_1^N}\left\{\mathbb{E}_{\underline{\hat{\gamma}}}\left[\sum_{i=1}^{N}\frac{L}{2T}\log(1+\hat{\gamma}_i p_i^*(\underline{\hat{\gamma}}))\right] \mid \mathbf{M}\right\} \tag{57}$$

$$\approx \mathbb{E}_{S_1^N}\left\{\mathbb{E}_{\underline{\hat{\gamma}}}\left[\sum_{i=1}^{N}\frac{L}{2T}\log(\hat{\gamma}_i p_i^*(\underline{\hat{\gamma}}))\right] \mid \mathbf{M}\right\} \tag{58}$$

$$= \mathbb{E}_{S_1^N}\left\{\sum_{i=1}^{N}\frac{L}{2T}\mathbb{E}_{\underline{\hat{\gamma}}}\left(\log\hat{\gamma}_i\right)\right\} + \mathbb{E}_{S_1^N}\left\{\mathbb{E}_{\underline{\hat{\gamma}}}\left[\sum_{i=1}^{N}\frac{L}{2T}\log(p_i^*(\underline{\hat{\gamma}}))\right] \mid \mathbf{M}\right\}. \tag{59}$$

We have $(\hat{\gamma}_i)_{i\in[\![1,N]\!]}$ are i.i.d., so:

$$\mathbb{E}_{S_1^N}\left\{\sum_{i=1}^{N}\frac{L}{2T}\mathbb{E}_{\underline{\hat{\gamma}}}\left(\log\hat{\gamma}_i\right)\right\} = \sum_{i=1}^{N}\frac{L}{2T}\mathbb{E}_{\hat{\gamma}_i}\left(\log\hat{\gamma}_i\right) \tag{60}$$

$$= \sum_{i=1}^{N}\frac{L}{2T}\mathbb{E}_{\hat{\gamma}}\left(\log\hat{\gamma}\right) \tag{61}$$

$$= \frac{\mathbb{E}_{\hat{\gamma}}\left(\log\hat{\gamma}\right)}{2}. \tag{62}$$

Because the $\log$ function is concave, so:

$$\mathbb{E}_{S_1^N}\left\{\mathbb{E}_{\underline{\hat{\gamma}}}\left[\sum_{i=1}^{N}\frac{L}{2T}\log(p_i^*(\underline{\hat{\gamma}}))\right] \mid \mathbf{M}\right\} \leq \mathbb{E}_{S_1^N}\left\{\sum_{i=1}^{N}\frac{L}{2T}\log(\mathbb{E}_{\underline{\hat{\gamma}}}\left[p_i^*(\underline{\hat{\gamma}})\right]) \mid \mathbf{M}\right\}.$$

Then (33) follows immediately.

Then, we have:

$$AT_{P.CSI-T} \leq \mathbb{E}_{S_1^N}\left\{\sum_{i=1}^{N}\frac{L}{2T}\log(\mathbb{E}_{\underline{\hat{\gamma}}}\left[p_i^*(\underline{\hat{\gamma}})\right]) \mid \mathbf{M}\right\} + \frac{\mathbb{E}_{\hat{\gamma}}\left(\log\hat{\gamma}\right)}{2} \tag{63}$$

Using Lemma 1

$$AT_{P.CSI-T} \leq \mathbb{E}_{S_1^N}\left\{\sum_{i=1}^{N}\frac{L}{2T}\log(p_{stat,i}^*) \mid \mathbf{M}\right\} + \frac{\mathbb{E}_{\hat{\gamma}}\left(\log\hat{\gamma}\right)}{2} \tag{64}$$

$$= AT_{Stat} + \frac{\mathbb{E}_{\hat{\gamma}}\left(\log\hat{\gamma}\right)}{2}. \tag{65}$$

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