# Capacity of Spectrum Sharing Cognitive Radio Systems over Nakagami Fading Channels at Low SNR

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***Abstract*—In this paper, we study the ergodic capacity of Cognitive Radio (CR) spectrum sharing systems at low power regime. We focus on Nakagami fading channels. We formally define the low power regime and present closed form expressions of the capacity in the low power regime under various types of interference and/or power constraints, depending on the available channel state information (CSI) of the cross link (CL) between the secondary user transmitter and the primary user receiver. We explicitly characterize two regimes where either the interference constraint or the power constraint dictates the optimal power profile. Our framework also highlights the effects of different fading parameters on the secondary link ergodic capacity. Interestingly, we show that the low power regime analysis provides a specific insight on the capacity behavior of CR that has not been reported by previous studies.**

***Index Terms*—Underlay Cognitive Radio, Ergodic Capacity, Spectrum Sharing, Low SNR, Nakagami fading, Interference and Power constraints.**

## I. Introduction

Due to the outstanding evolution in wireless communications technologies, the frequency spectrum became a scarce resource as most of the bandwidths are occupied. However, the use of this spectrum is not optimized. Consequently, a better spectrum management policy should be adopted in order to prevent spectrum saturation. Consequently, the Cognitive Radio (CR) concept, introduced by Mitola and Maguire [1], has been regarded as an efficient way to overcome this issue. Spectrum sharing or underlay CR is of particular interest due to its tremendous promising throughput [2]. A large body of work is now available and has addressed either fundamental limits of spectrum sharing protocol or/and practical schemes that can be deployed in real systems, e.g., [2]–[9]. However, to the best of our knowledge, no much work has focused on performance limits of such systems in the low power regime. Indeed, in many communication scenarios, the available power per degree of freedom can be vanishingly small [10], e.g., wideband systems, sensor networks and communication at the edge of cellular networks to cite a few. This has been a strong motivation for researchers to study this regime in order to better understand performance limits in this case [11]–[13]. Along similar lines of taught, we intend in this paper to shed some lights on spectrum sharing CR in the low power regime. Although this topic has been extensively investigated in the high-power regime, we believe that the low power framework highlights some important facts that can be opportunistically utilized in a wide range of applications. More precisely, in this paper, we focus on underlay CR, in which a secondary user shares the spectrum with a primary licensed user, where both users are constrained by stringent low power constraints. We are then curious to investigate the ergodic capacity of the secondary user at this regime. Although our focus is on Nakagami fading channels since it captures a large typical wireless links, our framework can be extended in a natural way to arbitrary fading channels, but with proper adaptations. To that end, our contribution in this paper is summarized as follows:

- We derive the spectrum sharing capacity under various interference constraints, in the asymptotically low power regime dictated by a low interference threshold, depending on the available channel state information (CSI), assuming first that there is no power restriction at the secondary transmitter. We show that the capacity is at least linear in the interference threshold.
- When a power constraint is introduced, we identify explicitly two extreme regimes in which the capacity depends only on either the power constraint or the interference constraint.
- We highlight the effect of cross link (CL) CSI on the low SNR capacity.

The remainder of the paper is organized as follows. In Section II, the system model is presented. In Section III, the capacity of low interference threshold with no power restrictions is derived. The low SNR capacity when a power constraint is introduced, is computed in Section IV. Selected numerical results along with their interpretations are presented in Section V. Finally, a conclusion of the work is given in Section VI.

## II. System Model

We consider a spectrum sharing model as illustrated in Fig. 1 where two users communicate with a base station through the same narrow-band frequency. The first user, called primary user (PU) is licensed to freely exploit the spectrum; whereas the second user, called secondary user (SU), is allowed to share the spectrum with the PU without

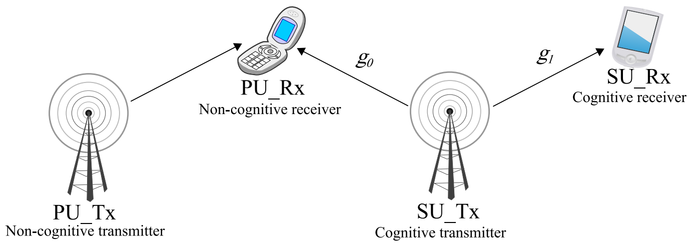


Figure 1. Spectrum sharing system model.

affecting the primary communication. We note by $g_0$ and $g_1$ the instantaneous channel gains between the secondary transmitter and both the primary receiver (PU_Rx) and the secondary receiver (SU_Rx), respectively [4]. These gains are assumed to be ergodic, independent, and their corresponding probability density function (p.d.f) $f_{g_0}(\cdot)$ and $f_{g_1}(\cdot)$ are continuous. We assume that the interference caused by the primary transmitter (PU_Tx) is treated as background noise at the secondary receiver. This assumption is reasonable if for instance the PU_Tx is far a part from the SU_Rx otherwise, this specific choice provides a lower bound on the performance [5]. Furthermore, we consider two types of constraints on the SU_Tx transmission; interference and/or power constraints [8]. The channels between the SU_Tx and the PU_Rx, and between the SU_Tx and the SU_Rx are called cross link and secondary link (SL), respectively. We assume a unit variance noise and a unit bandwidth, the capacity is presented in nats per channel use. In our model, we adopt two constraints, the first is an average transmit power constraint at the SU_Tx [8] describing the available power, the second is an interference constraint characterizing how tolerant the PU_Rx is toward violating an interference threshold by the SU. Depending on the available CSI of the Cross Link (CSI-CL), the interference constraint can be either a peak or an average interference constraint if full CSI-CL is available at the SU-Tx; or a statistical interference constraint if the CSI-CL is not available . We define the low power (also called low SNR for brevity) regime, as a scenario where the interference constraint and the transmit power constraint (when considered) are both asymptotically low, i.e., converge toward zero. We define $f(x) \approx g(x)$ if and only if $\lim_{x\to 0}\left(\frac{f(x)}{g(x)}\right) = 1$.

## III. Interference Constraint with No Power Restrictions

In a CR scenario, the framework of having only an interference constraint occurs when dealing with a non-limited power systems, such as base stations. In many cases, the tolerated interference is very low due to a strict primary user or a nearby communication between the primary and secondary users. Therefore, we investigate the effect of a very low interference threshold noted $I_{peak}$ on cognitive capacity with various levels of CSI-CL knowledge.

### A. Perfect CSI-CL

When full CSI-CL is available, two types of primary communication protection can be adopted to avoid the interference caused by the SU; peak and average interference constraints. We study their effect separately below.

*1) Peak Interference Constraint:* By fixing an adequate interference threshold $I_{peak}$, the SU is not allowed to exceed this threshold regardless of the channel condition. Note that this is always possible since the instantaneous channel gain is available at the Su_Tx. The capacity then is solution of the following problem

$$C = \max_{P(g_0,g_1)\geqslant 0} \mathbb{E}_{g_0,g_1}\left[\log\left(1 + P(g_0,g_1)\ g_1\right)\right], \tag{1}$$

$$\text{subject to} \quad P(g_0,g_1)\ g_0 \leqslant I_{peak}, \tag{2}$$

where $\mathbb{E}[\cdot]$ is the expectation operator. The solution of this problem is simple to derive and the optimal power is just given by $P^*(g_0,g_1) = \frac{I_{peak}}{g_0}$. For general Nakagami fading, we compute $\mathbb{E}_{g_0,g_1}[\frac{g_1}{g_0}]$ using the distribution of the ratio of two Nakagami random variables; $g_{10} = \frac{g_1}{g_0}$. The corresponding p.d.f can be easily derived [14] and it is given by

$$f_{g_{10}}(x) = \frac{(\frac{m_0}{\Omega_0})^{m_0}(\frac{m_1}{\Omega_1})^{m_1} x^{m_1-1}}{\left(\frac{m_0}{\Omega_0} + \frac{m_1 x}{\Omega_1}\right)^{m_0+m_1}\beta(m_0,m_1)}, \tag{3}$$

where $\beta(x,y)$ is the beta function defined by $\beta(x,y) = \frac{\Gamma(x)\Gamma(x)}{\Gamma(x+y)}$ and $\Gamma(\cdot)$ is the gamma function defined by $\Gamma(x) = \int_0^\infty e^{-t}t^{x-1}dt$. Equation (3) yields a capacity that has three expressions depending on the parameter $m_0$. The following regimes are computed after using the series expansion of $I_{peak}$ near zero:

$$\begin{cases} \text{If } \frac{1}{2} < m_0 < 1, \text{ then } C(I_{peak}) \approx \left(I_{peak}\frac{\Omega_1}{\Omega_0}\right)^{m_0} \frac{m_0^{m_0-1}\pi}{m_1^{m_0}\beta(m_0,m_1)\sin(\pi m_0)}, \\ \text{If } m_0 = 1, \text{ then } C(I_{peak}) \approx I_{peak}\frac{\Omega_1}{\Omega_0}\log(\frac{1}{I_{peak}}), \\ \text{If } 1 < m_0, \text{ then } C(I_{peak}) \approx I_{peak}\frac{\Omega_1}{\Omega_0}\frac{m_0\Gamma(m_0-1)}{\Gamma(m_0)}. \end{cases} \tag{4}$$

The results in (4) show that $m_0$ is a key parameter that defines the capacity behavior at low $I_{peak}$. Indeed, the capacity is linear in $I_{peak}\frac{\Omega_1}{\Omega_0}$ for $m_0 > 1$ and sub-linear for $\frac{1}{2} < m_0 < 1$, whereas for $m_0 = 1$, it scales essentially as $I_{peak}\log(\frac{1}{I_{peak}})$. Note also that as $I_{peak} \to 0$, the capacity progressively increases as $m_0$ decreases.

*2) Average Interference Constraint:* A second alternative to protect the primary communication when full CSI-CL is available, is to impose an average interference constraint on the SU_Tx. This constraint is less stringent than the instantaneous interference. Subsequently, the ergodic capacity is computed by solving the following optimization problem:

$$C = \max_{P(g_0,g_1)\geqslant 0} \mathbb{E}_{g_0,g_1}\left[\log\left(1 + P(g_0,g_1)g_1\right)\right], \tag{5}$$

$$\text{subject to} \quad \mathbb{E}_{g_0,g_1}\left[P(g_0,g_1)\ g_0\right] \leqslant I_{peak}. \tag{6}$$

Using the Lagrangian method, the optimal power is given by the water-filling expression as in [4] yielding

$$P^*(g_0,g_1) = \left(\frac{1}{\lambda\ g_0} - \frac{1}{g_1}\right)^+, \tag{7}$$

where $\left(\cdot\right)^{+} = \max(0, \cdot)$ and $\lambda$ is the Lagrange multiplier, computed by solving the constraint with equality i.e. :

$$\mathbb{E}_{g_0,g_1}\left[\left(\frac{1}{\lambda} - \frac{g_0}{g_1}\right)^{+}\right] = I_{peak}. \tag{8}$$

Let $g_{10} = \frac{g_1}{g_0}$ and $f_{g_{10}}(\cdot)$ its p.d.f, the equality in (8) may be written as:

$$\int_{g_{10}>\lambda} \left(\frac{1}{\lambda} - \frac{1}{g_{10}}\right) f_{g_{10}}(g_{10})\ d\ g_{10} = I_{peak}. \tag{9}$$

Likewise, the corresponding capacity is given by:

$$C = \int_{g_{10}>\lambda} \log\left(\frac{g_{10}}{\lambda}\right) f_{g_{10}}(g_{10})\ d\ g_{10}. \tag{10}$$

From (8), we notice that $\lambda$ is inversely proportional to $I_{peak}$. That is, as $I_{peak} \to 0$, $\lambda$ goes to infinity. Consequently, we compute the capacity at low SNR using (9) and (10) by performing series expansion as $\lambda \to \infty$ to obtain:

$$I_{peak} \approx \frac{1}{m_0(m_0+1)\ \beta(m_0, m_1)} \left(\frac{m_0\Omega_1}{m_1\Omega_0}\right)^{m_0} \left(\frac{1}{\lambda}\right)^{m_0+1}, \tag{11}$$

$$C \approx \frac{1}{m_0^2\ \beta(m_0, m_1)} \left(\frac{m_0\Omega_1}{m_1\Omega_0}\right)^{m_0} \left(\frac{1}{\lambda}\right)^{m_0}, \tag{12}$$

By expressing $\frac{1}{\lambda}$ as a function of $I_{peak}$ in (11) and inserting the result in (12), we obtain

$$C(I_{peak}) \approx \left(I_{peak}\right)^{\frac{m_0}{1+m_0}} \frac{(m_0(m_0+1))^{\frac{m_0}{1+m_0}}}{m_0^2\ \beta(m_0, m_1)^{\frac{1}{1+m_0}}} \left(\frac{m_0\Omega_1}{m_1\Omega_0}\right)^{\frac{m_0}{1+m_0}}. \tag{13}$$

Note that in the particular case where $m_0 = m_1 = m$ and $\Omega_0 = \Omega_1 = \Omega$, the capacity becomes

$$C(I_{peak}) \approx \left(I_{peak}\right)^{\frac{m}{1+m}} \frac{(m(m+1))^{\frac{m}{1+m}}}{m^2\ \beta(m, m)^{\frac{1}{1+m}}}. \tag{14}$$

We note that, the capacity in (14) scales essentially as $\left(I_{peak}\right)^{\frac{m}{1+m}}$ which is much higher than $I_{peak}$, at asymptotically low $I_{peak}$, i.e., $\left(I_{peak}\right)^{\frac{m}{1+m}} \gg I_{peak}$. If, in addition, $m_0 = m_1 = 1$ (this corresponds to a situation where both the SL and the CL undergo a Rayleigh fading) the corresponding capacity at low $I_{peak}$ simplifies to

$$C(I_{peak}) \approx \sqrt{2\ I_{peak}\ \frac{\Omega_1}{\Omega_0}}. \tag{15}$$

### *B. No CSI-CL with Statistical Interference Constraint*

When the CSI-CL is not available, the primary communication is protected through a statistical constraint in which the PU tolerates that the SU exceeds the interference threshold with a certain probability lower than a threshold $\varepsilon$. The problem can be formulated as follows:

$$C = \max_{P(g_1)\geqslant 0} \mathbb{E}_{g_0,g_1}\left[\log\left(1 + P(g_1)g_1\right)\right], \tag{16}$$

$$\text{subject to} \quad \text{Prob}\left\{P(g_1)g_0 \geqslant I_{peak}\right\} \leqslant \varepsilon. \tag{17}$$

Note that (17) can be written as in [15] as:

$$P(g_1) \leqslant P_{peak}, \tag{18}$$

where $P_{peak} = \frac{I_{peak}}{F_{g_0}^{-1}(1-\varepsilon)}$ and $F_{g_0}^{-1}(\cdot)$ is the inverse of the cumulative density function of $g_0$ given by $F_{g_0}(x) = 1 - \frac{\Gamma(m_0, \frac{m0x}{\Omega_0})}{\Gamma(m_0)}$, where $\Gamma(\cdot,\cdot)$ is the incomplete Gamma function. The solution of this problem is, again, simple to find since the objective function is convex and the constraint is linear. The optimal power is given by $P^{*}(g_1) = P_{peak}$. The corresponding capacity is given by:

$$C(I_{peak}) \approx \mathbb{E}_{g_1}[P_{peak}g_1] = I_{peak}\frac{\Omega_1}{F_{g_0}^{-1}(1-\varepsilon)}, \tag{19}$$

which is only linear in $I_{peak}$.

## IV. Introduction Of A Power Constraint

In many mobile applications, the power is a crucial parameter that the systems has to manage optimally to efficiently perform the communications. Consequently, in addition to the interference constraint, we introduce an average power constraint given by:

$$\mathbb{E}_{g_0,g_1}\left[P(g_0, g_1)\right] \leqslant P_{avg}, \tag{20}$$

where $P_{avg}$ is the available power at the secondary transmitter. In the low SNR regime, i.e $P_{avg} \to 0$, the corresponding capacity was studied in a non-cognitive context in [16] and is shown to be equal to

$$C(P_{avg}) \approx \frac{\Omega_1}{m_1}\ P_{avg}\ \log\left(\frac{1}{P_{avg}}\right). \tag{21}$$

Nevertheless, due to the interference threshold $I_{peak}$ which is low too, the capacity may depend on both $P_{avg}$ and $I_{peak}$. In this section, we analyze the capacity under peak interference constraints with perfect CSI. By taking into consideration the power constraint, the capacity is now solution of the following problem

$$C = \max_{P(g_0,g_1)\geqslant 0} \mathbb{E}_{g_0,g_1}\left[\log\left(1 + P(g_0, g_1)g_1\right)\right], \tag{22}$$

$$\text{subject to} \quad P(g_0, g_1)g_0 \leqslant I_{peak}, \tag{23}$$

$$\mathbb{E}_{g_0,g_1}\left[P(g_0, g_1)\right] \leqslant P_{avg}. \tag{24}$$

The solution of this problem has been derived in [17] and is given by:

$$P^{*}(g_0, g_1) = \min\left\{\left(\frac{1}{\lambda} - \frac{1}{g_1}\right)^{+}, \frac{I_{peak}}{g_0}\right\}, \tag{25}$$

where $\lambda$ is the Lagrange multiplier that satisfies the power constraint with equality, i.e., $\mathbb{E}_{g_0,g_1}\left[P^{*}(g_0, g_1)\right] = P_{avg}$. The corresponding capacity presents, mainly, two regimes: power constrained regime (i.e. $C = f(P_{avg})$) and interference constrained regime (i.e. $C = f(I_{peak})$). In the power constrained regime the interference constraint is not "felt" by the SU_Tx and, hence, does not affect the optimal power, and the capacity is the same as in a non-cognitive scenario [16]. However, in some cases, the interference threshold is also very low, i.e $I_{peak} \to 0$ which requires a new analyze in order to find the low SNR capacity. Note that the transition between these two regimes corresponds to a capacity depending on the combination of the two constraint parameters. In order to characterize the capacity regimes, let us first split the power profile in (25), as in [18]:

$$\begin{cases} \text{if } P_{avg} > \mathbb{E}_{g_0,g_1}\left[\frac{I_{peak}}{g_0}\right], \text{ then } P^*(g_0,g_1) = \frac{I_{peak}}{g_0}, \\ \text{if } \mathbb{E}_{g_0,g_1}\left[\frac{I_{peak}}{g_0} - \frac{1}{g_1}\right] < P_{avg} < \mathbb{E}_{g_0,g_1}\left[\frac{I_{peak}}{g_0}\right] \\ \qquad \text{then } P^*(g_0,g_1) = \min\left\{\left(\frac{1}{\lambda} - \frac{1}{g_1}\right)^+, \frac{I_{peak}}{g_0}\right\}, \\ \text{if } P_{avg} < \mathbb{E}_{g_0,g_1}\left[\frac{I_{peak}}{g_0} - \frac{1}{g_1}\right], \text{ then } P^*(g_0,g_1) = \left(\frac{1}{\lambda} - \frac{1}{g_1}\right)^+. \end{cases} \tag{26}$$

For convenience, let us define the function $H(x)$ as

$$H(x) = \mathbb{E}_{g_1}\left[\left(x - \frac{1}{g_1}\right)^+\right].$$

Analyzing more closely (26), we obtain

$$\text{If } P_{avg} < \mathbb{E}_{g_0}\left[H\left(\frac{I_{peak}}{g_0}\right)\right] \text{the capacity is given by (21).} \tag{27}$$

$$\text{If } P_{avg} > \mathbb{E}_{g_0}\left[\frac{I_{peak}}{g_0}\right], \text{ i.e. } P_{avg} > I_{peak}\frac{m_0\Gamma(m_0-1)}{\Omega_0\Gamma(m_0)}, \text{ for } m_0 > 0 \tag{28}$$

then the capacity is given by (4). Note that for the case of $\frac{1}{2} < m_0 \leq 1$, we have $\mathbb{E}_{g_0}\left[\frac{I_{peak}}{g_0}\right] \to \infty$. Hence, the condition, $P_{avg} > \mathbb{E}_{g_0}\left[\frac{I_{peak}}{g_0}\right]$ is never satisfied and the capacity goes asymptotically to the expression in (4).

## V. Numerical Results

In Fig. 2, the SU capacity under an average interference constraint is plotted in nats per channel use (npcu) as a function of the interference threshold $I_{peak}$. In Fig. 2, we set $m_0 = m_1 = 2$ in order to analyze the effect of each channel mean. As shown in Fig. 2, the capacity and its asymptotic approximation given by (13) are indistinguishable for low values of $I_{peak}$. Moreover, as depicted in Figure 2, as the mean of the cross link, $\Omega_0$, increases, the capacity decreases and increasing the channel gain $\Omega_1$ enhances the capacity.

When the SL and the CL have equal fading parameter $m$ ($m_0 = m_1 = m$), then for a given $I_{peak}\frac{\Omega_0}{\Omega_1}$, the capacity decreases with $m$ as depicted in Fig. 3. Indeed, this is expected and is in agreement with the related asymptotic capacity given by (14), also shown in Fig. 3. Fig. 4 shows the effect of different interference constraints on the capacity as a function of $I_{peak}$ for $m_0 = m_1 = 2$ and $\Omega_0 = \Omega_1 = 1$. As expected, the statistical constraint gives the lowest capacity since the CSI-CL is not available and no power adaptation is performed. The availability of CSI-CL improves considerably the capacity even under a peak interference constraint. The improvement is huge if instead, a more relaxed average interference constraint is adopted. To achieve this huge improvement, power adaptation is necessary. This is again in full agreement with our analysis since the exponent of $I_{peak}$ in the average constrained capacity given by (14) is equal to $\frac{m}{m+1}$, whereas that of the peak constrained capacity given by (4) is equal to 1. We note that these observations have not been pointed out by previous studies on CR literature, as they are not necessarily focusing on low power regime. Indeed, it takes an asymptotic analysis

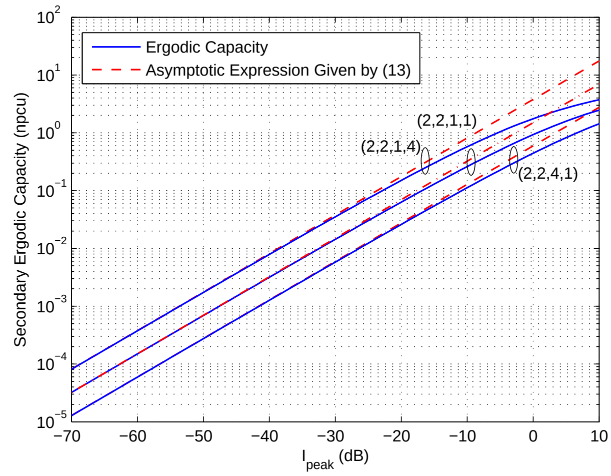


Figure 2. Capacity versus $I_{peak}$ for different values of $m_0, m_1, \Omega_0$ and $\Omega_1$

to gain such an insight.

In Fig. 5, the effect of the interference and the average power constraints is highlighted on the low SNR capacity for the case of $m_0 = m_1 = 1$ and $\Omega_0 = \Omega_1 = 1$. The capacity versus $P_{avg}$ is plotted for different values of $I_{peak}$ under peak interference constraint. As can be seen in Fig. 5, there exists a saturation of the capacity curve as $P_{avg}$ goes to high values. The beginning of this regime is earlier for lower values of $I_{peak}$. Meanwhile, the non-saturated capacity regime present a setting in which both PU and SU are using their full power without harming each other.

## VI. Conclusion

In this paper, we have studied the ergodic capacity of the secondary link in a spectrum sharing protocol at low power regime. Although, we have focused on Nakagami fading channels, we argue that our framework can be generalized to account for different fading statistics. First, we have considered a setting where the CR user is solely constrained by a low interference threshold. We have shown that in this setting, the capacity scales at least linearly with the interference threshold. Then, we have introduced, in addition to the interference constraint, a transmit power constraint. Beyond the insightful simple closed form expressions of the capacity at low power regime that have been presented in this paper, we have identified certain regimes where coexistence of a CR user along with a primary user is “harmonious” in the sense that no one is hurt by the presence of the other.

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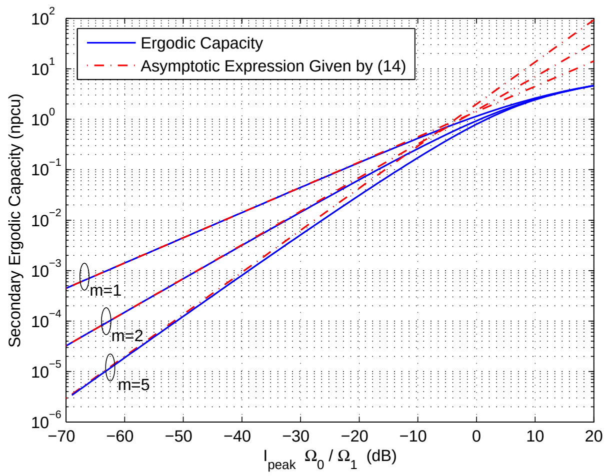


Figure 3. Capacity versus $I_{peak}\frac{\Omega_0}{\Omega_1}$ for different values of m.

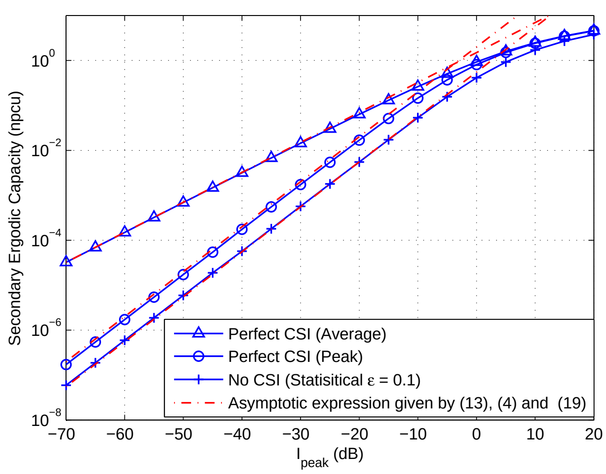


Figure 4. Capacity versus $I_{peak}$ for different interference constraint types, $m_0 = m_1 = 2$ and $\Omega_0 = \Omega_1 = 1$.

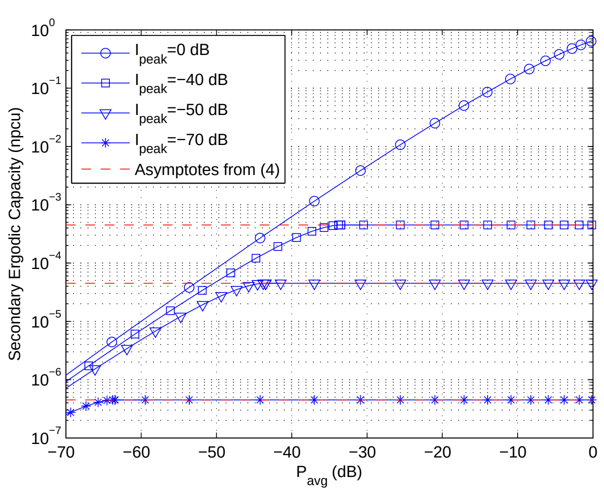


Figure 5. Capacity versus $P_{avg}$ for different values of $I_{peak}$ under peak interference constraint.


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