Achievable Rate of Cognitive Radio Spectrum Sharing MIMO Channel with Space Alignment and Interference Temperature Precoding

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Abstract*—In this paper, we investigate the spectral efficiency gain of an uplink Cognitive Radio (CR) Multi-Input Multi-Output (MIMO) system in which the Secondary/unlicensed User (SU) is allowed to share the spectrum with the Primary/licensed User (PU) using a specific precoding scheme to communicate with a common receiver. The proposed scheme exploits at the same time the free eigenmodes of the primary channel after a space alignment procedure and the interference threshold tolerated by the PU. In our work, we study the maximum achievable rate of the CR node after deriving an optimal power allocation with respect to an outage interference and an average power constraints. We, then, study a protection protocol that considers a fixed interference threshold. Applied to Rayleigh fading channels, we show, through numerical results, that our proposed scheme enhances considerably the cognitive achievable rate. For instance, in case of a perfect detection of the PU signal, after applying Successive Interference Cancellation (SIC), the CR rate remains non-zero for high Signal to Noise Ratio (SNR) which is usually impossible when we only use space alignment technique. In addition, we show that the rate gain is proportional to the allowed interference threshold by providing a fixed rate even in the high SNR range.

***Index Terms*—Achievable Rate, Underlay Cognitive Radio, Spectrum Sharing, Space alignment Interference Alignment, Successive Interference Cancellation.**

I. Introduction

Due to the spread of the current wireless services and wireless communication evolution, more bandwidth is needed in order to offer more services with high data rates. Consequently, the accessible radio spectrum is becoming critically scarce as mentioned the Federal Communications Commission (FCC) [1]. To overcome this shortage, current spectrum allocation policy, relatively inefficient, should be substituted by an optimized spectrum management concept that avoids unused spectrum holes. In this vision, the Cognitive Radio (CR) concept was introduced by Mitola [2] in order to optimize the use of the spectrum within multiple users. The main idea is to allow secondary (non-licensed /cognitive) users, noted ”SU”, to share the spectrum with the primary (licensed / non cognitive) users , noted ”PU”, without affecting the primary communication. During last years, many CR techniques where introduced [3]: underlay, overlay, and interweave. Meanwhile, we distinguish two CR strategies:

- Orthogonal transmission: in which the PU does not ”feel” the SU communication at all. The SU spots spectrum holes in (space, time and frequency) then opportunistically performs communication. This strategy is called: interweave CR mode [4].
- Non-orthogonal transmission: in which the SU is allowed to communicate simultaneously with the PU. However, the secondary communication is limited to a certain interference-temperature tolerated by the primary user [5], [6]. This strategy is called underlay CR mode.

In this paper, we investigate the combination of both orthogonal and non-orthogonal CR transmission modes in an uplink Multiple-Input and Multiple-Output (MIMO) antenna communication. Our objective is to examine the maximum achievable rate for the SU over all channel realizations. Adopting Multiple Input Multiple Output (MIMO) power allocation within a CR framework was widely studied in [7]–[10]. Differently from previous works, we present a new precoding and decoding scheme allowing the SU to achieve higher rate with minimal effect on the PU communication. Meanwhile, authors in [11] are adopting a precoding scheme of uplink MIMO but in a non-CR context. The SU maximizes its rate, by allocating optimally its power among its antennas depending on the communication environment including the primary communication activity. In our setting, after a special precoding in the PU transmitter, some free eigenmodes are unused which could be freely exploited by the SU. Moreover, the SU exploits also the non-free eigenmodes with respect to an interference threshold defined by the PU. At the common receiver, the PU signal transmitted through the exploited eigenmodes is decoded first. Once, the effect of the PU signal is eliminated from the received signal using a Successive Interference Cancellation (SIC) decoder [12], the SU signal is decoded. In addition, we study the SU rate depending on the PU signal detection accuracy. We, also, study a primary communication protection technique with fixed interference threshold. The rest of this paper is organized as follows. In Section II, the system model is presented. Section III describes the precoding and decoding strategies. Achievable rate expressions of the SU are derived with fixed interference threshold in Section IV. Numerical results are presented in Section V. Finally, the paper is concluded in Section VI.

II. System Model

We consider an uplink communication system consisting of two transmitters "PU" and "SU" and a common receiver noted "R". The PU, as a licensed user, exploits the channel while the SU, as an unlicensed node, is allowed to share opportunistically the spectrum and to access the channel under some constraints to maintain a certain Quality of Service (QoS) of the primary communication. We assume that each node is equipped with N antennas, and the channel gain matrices representing the links between the PU and R (PU-R) and between SU and R (SU-R) are denoted by $\boldsymbol{H_{pp}}$ and $\boldsymbol{H_{sp}}$, respectively, as shown in Fig. 1. In the framework of an uplink scenario where the receiver is common to both transmitters, the interference may cause a significant deterioration to both primary and secondary performances. However, by adopting an interference temperature protection [13], the PU tolerates an interference level below a certain threshold defined for each receive antenna. That is, if the SU satisfies this constraint, we consider that the QoS of the PU is not affected (i.e. the PU is able to achieve its required rate). On the other hand, we exploit the interference alignment technique presented in [8] to enable the SU to access the channel without causing any interference to the PU by applying a simple linear precoding described in Section III. We assume that full Channel State Information (CSI) is available at the receiver and the secondary transmitter (i.e PU-R and SU-R channel gains). Meanwhile, the primary receiver is assumed to only know the PU-R CSI. The received signal $\boldsymbol{y}$ at the receiver, R, is expressed as follows

$$\boldsymbol{y} = \boldsymbol{H_{pp}}\boldsymbol{\Phi_p}\boldsymbol{s_p} + \boldsymbol{H_{sp}}\boldsymbol{\Phi_s}\boldsymbol{s_s} + \boldsymbol{w}, \quad (1)$$

where $\boldsymbol{H_{pp}}$ and $\boldsymbol{H_{sp}}$ are assumed to be independent and identically distributed (i.i.d.), $\boldsymbol{\Phi_p}$ and $\boldsymbol{\Phi_s}$ are the linear precoding matrices applied at the PU and SU, respectively. Meanwhile, $\boldsymbol{s_p}$ and $\boldsymbol{s_s}$ denote the signal transmitted by the PU and SU, respectively. For $i \in \{p, s\}$ we consider $\boldsymbol{P_i} = \mathbb{E}[\boldsymbol{s_i}\boldsymbol{s_i}^h]$ to be the covariance matrix of the vector $\boldsymbol{s_i}$, where $\mathbb{E}[\cdot]$ is the expectation operator. This covariance matrix is subject to a power constraint $Tr\ (\boldsymbol{P_i}) \leq P_{tot}$ where $Tr\ (\boldsymbol{A}) = \sum_j A(j, j)$ is the trace of the matrix $\boldsymbol{A}$, and P_{tot} is the total power budget considered to be the same for both users. Finally, $\boldsymbol{w}$ indicates a zero mean Additive White Gaussian Noise (AWGN) vector at the receiver with an identity covariance matrix; $\boldsymbol{I_N}$.

III. Precoding and Decoding Strategy

In this section, we present the proposed linear precoding and decoding techniques in order to maximize the rate of the SU without degrading the PU QoS by respecting the interference constraint. At the same time, we introduce the interference alignment technique presented in [8] which permits to the SU to transmit through the unused primary eigenmodes. In fact, by having a perfect CSI of the PU-R link at the PU transmitter, the PU can optimally allocate its power to maximize the achievable rate through a waterfilling policy. By applying a Singular Value Decomposition (SVD) to $\boldsymbol{H_{pp}}$, the PU transmits through parallel channels characterized by the associated eigenmodes. The SVD of the matrix is denoted $\boldsymbol{H_{pp}} = \boldsymbol{U}\boldsymbol{\Lambda}\boldsymbol{V}^h$ where $\boldsymbol{U}$ and $\boldsymbol{V}$ are two unitary matrices and $\boldsymbol{\Lambda}$ is a diagonal matrix that contains the ordered singular values of $\boldsymbol{H_{pp}}$ denoted as $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_N$. Note that the eigenvalues of $\boldsymbol{H_{pp}}$ are equivalent to the squares of its singular values as it is a square matrix. To transform the MIMO channel to N parallel channels, we perform linear precoding and decoding at the transmitter and at the receiver, respectively. Let

$$\boldsymbol{\Phi_p} = \boldsymbol{V} \text{ and } \boldsymbol{\Psi} = \boldsymbol{U}, \quad (2)$$

then, the output received signal becomes:

$$\boldsymbol{r} = \boldsymbol{\Psi}^h\boldsymbol{y} = \boldsymbol{\Lambda}\boldsymbol{s_p} + \boldsymbol{U}^h\boldsymbol{H_{sp}}\boldsymbol{\Phi_s}\boldsymbol{s_s} + \tilde{\boldsymbol{w}}, \quad (3)$$

where $\tilde{\boldsymbol{w}} = \boldsymbol{\Psi}^h\boldsymbol{w}$ remains a zero mean AWGN with a covariance matrix $\boldsymbol{I_N}$. Being a licensed user, the PU ignores the existence of SU and applies a waterfilling power allocation (WPA) [8] to maximize its achievable rate C_p without considering the interference effect by solving the following optimization problem:

$$\begin{aligned} \underset{\boldsymbol{P_p}}{\text{maximize}} \quad & C_p = \mathbb{E}\left[\log_2\left(det\left(\boldsymbol{I_N} + \boldsymbol{P_p}\boldsymbol{\Lambda}\boldsymbol{\Lambda}^h\right)\right)\right] \\ \text{subject to} \quad & Tr\ (\boldsymbol{P_p}) \leq P_{tot}. \end{aligned} \quad (4)$$

The solution of this WPA problem is given in [14] by:

$$P_p(j, j) = \left[\mu_0 - \frac{1}{\lambda_j{}^2}\right]^+ \text{ for } j = 1 \ldots N, \quad (5)$$

where $[.]^+ = max(0, .)$ and μ_0 is the Lagrangian multiplier corresponding to the primary average power constraint.

Note that in some cases, when the channel gain is poor, the number of used eigenmodes by the PU can be less than N when $\mu_0 - \frac{1}{\lambda_j^2} \leq 0$ for some j. Consequently, the unused eigenmodes could be freely exploited by SU. Let n $(0 \leq n < N)$ to be the number of unused eigenmodes, we distinguish two sets of eigenmodes: $N - n$ eigenmodes used by the PU and n unused eigenmodes that can be exploited by the SU.

In order to totally eliminate the effect of interference, an appropriate choice of $\boldsymbol{\Phi_s}$ has been proposed in [8] as $(\boldsymbol{H_{sp}})^{-1}\boldsymbol{U}\bar{\boldsymbol{P}}_{\boldsymbol{p}}$,

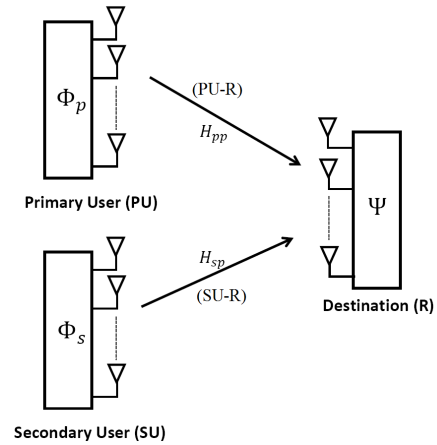

Figure 1. Uplink configuration for a cognitive radio system with N antennas.

where $\bar{\boldsymbol{P}}_{\boldsymbol{p}}$ is a diagonal matrix with the following entries,:

$$\bar{P}_p(j,j) = \begin{cases} 1 \text{ if } P_p(j,j) = 0 \\ \\ 0 \text{ if } P_p(j,j) \neq 0, \end{cases} \quad \text{for } j = 1 \ldots N \tag{6}$$

This choice of $\boldsymbol{\Phi}_{\boldsymbol{s}}$ prevents the SU to share the used eigenmodes and only use the free ones. In our framework, however, we allow the SU to transmit in all the eigenmodes by respecting a certain interference temperature threshold I_{peak} when sharing the used eigenmodes. Consequently, we choose $\boldsymbol{\Phi}$ as follows:

$$\boldsymbol{\Phi}_{\boldsymbol{s}} = (\boldsymbol{H}_{\boldsymbol{sp}})^{-1}\boldsymbol{U}. \tag{7}$$

Consequently, the output received signal becomes:

$$\boldsymbol{r} = \begin{bmatrix} \tilde{\boldsymbol{\Lambda}} & 0 \\ 0 & 0 \end{bmatrix}_{N\times N} \begin{bmatrix} \boldsymbol{s_p}^{(1)} \\ 0 \end{bmatrix}_{N\times 1} + \begin{bmatrix} \boldsymbol{s_s}^{(1)} \\ \boldsymbol{s_s}^{(2)} \end{bmatrix}_{N\times 1} + \begin{bmatrix} \tilde{\boldsymbol{w}}^{(1)} \\ \tilde{\boldsymbol{w}}^{(2)} \end{bmatrix}_{N\times 1} \tag{8}$$

where $\tilde{\boldsymbol{\Lambda}} = \text{diag}\,[\lambda_1 \cdots \lambda_{N-n}]$ (i.e. the $N-n$ non-zeros eigenvalues of $\boldsymbol{\Lambda}$), $\boldsymbol{s_p}^{(1)}$ is a vector composed by the $N-n$ transmitted PU signal through the $N-n$ first antennas, $\boldsymbol{s_s}^{(1)}$ and $\boldsymbol{s_s}^{(2)}$ correspond to the vectors composed by the transmitted SU signal through the $N-n$ used and n unused eigenmodes, respectively. The received signal is expressed as

$$r_j = \lambda_j s_{p_j} + s_{s_j} + \tilde{w}_j, \;\text{ for } j = 1 \cdots N-n, \tag{9}$$

$$r_j = s_{s_j} + \tilde{w}_j, \;\text{ for } j = N-n+1 \cdots n. \tag{10}$$

The detection of the SU signal is performed after applying an ordered SIC to estimate the strongest signal s_p. Note that it is natural to consider that $\boldsymbol{s_p}$, the signal from a licensed user, is, in average, stronger than the constrained SU signal. Note also that the SU signal transmitted over the n free eigenmodes is only constrained by a total power constraint.

IV. Rate of the Secondary User with fixed I_{peak}

In this section, we analyze the achievable rate of SU using the proposed technique depending on the perfectness of the SIC operation. We, first, analyze the performance of the system when a fully successful SIC is applied. We, then investigate the gain in performance with a degenerate SIC (i.e fully erroneous). We introduce a parameter α $(0 \le \alpha \le 1)$ that corresponds to the probability of detecting the PU signal $\boldsymbol{s_p}$ correctly before applying the SIC.

A. Perfect SIC

We consider that the PU message is always decoded perfectly. That is; $\hat{s}_{p_j} = s_{p_j}$ $\forall j = 1 \cdots N-n$ where $\hat{s}_{p_j}$ is the estimated PU signal at the jth receiving antenna. Hence, the cancellation of the PU effect is performed correctly ($\alpha = 1$) and, in this case, the output received signal after the SIC decoding, $\tilde{\boldsymbol{r}}$, is written as

$$\tilde{\boldsymbol{r}} = \boldsymbol{r} - \boldsymbol{\Lambda}\hat{\boldsymbol{s}}_{\boldsymbol{p}} = \boldsymbol{s_s} + \tilde{\boldsymbol{w}}. \tag{11}$$

Note that, the secondary channel becomes unitary due to the specific precoding described in (7) which provides a throughput independent of the secondary channel gain. Consequently, the corresponding achievable rate $C_s{}^{(\alpha=1)}$ is given by

$$\max_{\boldsymbol{P_s}} C_s{}^{(1)} = \sum_{j=1}^{N} \mathbb{E}\left[\log_2\left(1 + P_s(j,j)\right)\right]. \tag{12}$$

Thus, to find the optimal power allocation $\boldsymbol{P_s}^*$, we have to solve the following optimization problem:

$$\begin{aligned} &\max_{\boldsymbol{P_s}} C_s{}^{(1)} = \sum_{j=1}^{N} \mathbb{E}\left[\log_2\left(1 + P_s(j,j)\right)\right] \\ &\text{subject to } \; Tr\;(\boldsymbol{P_s}) \le P_{tot} \\ &\text{and } \; P_s(j,j) \le I_{peak}(j) \text{ for } j = 1 \cdots N-n. \end{aligned} \tag{13}$$

We solve this problem using the Lagrange method [15] and the resulting power profile is formulated in the following theorem.

Theorem

The solution to the optimization problem defined by (13) is

$$P_s^*(j,j) = \begin{cases} \min\left\{[\frac{1}{\mu} - 1]^+, I_{peak}(j)\right\} \text{ for } j = 1 \cdots N-n, \\ [\frac{1}{\mu} - 1]^+ \quad \text{for } j = N-n+1 \cdots N, \end{cases} \tag{14}$$

where μ is the lagrange multiplier associated to the average power constraint.

Proof: The proof is presented in the Appendix. ∎

Note that, the optimal power does not depend on the primary transmission but only on I_{peak} and P_{tot} through μ.

B. Degenerate SIC

In this case, we assume that, the interference cancellation operation is totaly wrong ($\alpha = 0$). The output received signal can be written as follows

$$\tilde{r} = \begin{bmatrix} \tilde{\boldsymbol{\Lambda}} & 0 \\ 0 & 0 \end{bmatrix}_{N\times N} \begin{bmatrix} \boldsymbol{s_p}^{(1)} - \hat{\boldsymbol{s}}_{\boldsymbol{p}}^{(1)} \\ 0 \end{bmatrix}_{N\times 1} + \begin{bmatrix} \boldsymbol{s_s}^{(1)} \\ \boldsymbol{s_s}^{(2)} \end{bmatrix}_{N\times 1} + \begin{bmatrix} \tilde{\boldsymbol{w}}^{(1)} \\ \tilde{\boldsymbol{w}}^{(2)} \end{bmatrix}_{N\times 1}, \tag{15}$$

where $\hat{\boldsymbol{s}}_{\boldsymbol{p}}^{(1)}$ is a wrong estimate of $\boldsymbol{s_p}^{(1)}$. We investigate, in this extreme condition, the achievable rate of the SU by assuming that the decoded error is independent of the signal that has been actually transmitted. Furthermore, since $\hat{\boldsymbol{s}}_{\boldsymbol{p}}$ and $\boldsymbol{s_p}$ belong to the same constellation, then $\mathbb{E}\left[(\boldsymbol{s_p} - \hat{\boldsymbol{s}}_{\boldsymbol{p}})|^2\right] = 2\boldsymbol{P_p}$. Thus, we have to solve the following optimization problem:

$$\begin{aligned} \max_{\boldsymbol{P_s}} C_s{}^{(0)} &= \sum_{j=1}^{N-n} \mathbb{E}\left[\log_2\left(1 + \frac{P_s(j,j)}{1 + 2P_p(j,j)\lambda_j{}^2}\right)\right] \\ &+ \sum_{j=N-n+1}^{N} \mathbb{E}\left[\log_2\left(1 + P_s(j,j)\right)\right] \end{aligned} \tag{16}$$

$$\begin{aligned} &\text{subject to } \; Tr\;(\boldsymbol{P_s}) \le P_{tot} \\ &\text{and } \; P_s(j,j) \le I_{peak}(j) \text{ for } j = 1 \cdots N-n \end{aligned} \tag{17}$$

In this scenario the optimal power is computed similarly to the perfect SIC case by using the Lagrange method, the optimal power is given by

$$P_s^*(j,j) = \begin{cases} \min\left\{[\frac{1}{\mu} - (1 + 2P_p(j,j)\lambda_j{}^2)]^+, I_{peak}(j)\right\} \\ \text{ for } j = 1 \cdots N-n, \\ [\frac{1}{\mu} - 1]^+ \quad \text{for } j = N-n+1 \cdots N, \end{cases}$$

where μ is the lagrange multiplier associated to the average power constraint. We notice, here, that the optimal power depends on the primary power and eigenmodes which means that the secondary is adapting its power continuously with the variation of the primary channel state.

V. Numerical Results

In this section, we provide numerical results for the SU rate using our precoding scheme. We adopt a Rayleigh fading channel in which the entries of $\boldsymbol{H_{pp}}$ and $\boldsymbol{H_{sp}}$ are complex Gaussian random variables with zero mean and unit variance (noted $\mathcal{N}(0;1)$).

In Figure 2, we plot the SU achievable rate as a function of P_{tot} for $N = 4$ antenna when the SIC is perfectly performed. The free eigenmodes (FE) and interference temperature (IT) precoding rate are plotted along with: i) a precoding with only Free Eigenmodes Power Allocation (FE-PA) and ii) a Uniform Power Allocation (UPA) with no the PU. Figure 2 shows that the interference tolerance allows the SU to achieve better rates when P_{tot} is greater than 5 dB. In addition, as P_{tot} increases, the achievable rate reaches a maximum before decreasing to lower asymptotic values, for fixed I_{peak}. This variation in the secondary rate is related to both the variation of the number of the free eigenmodes n and the secondary channel condition. In fact at low P_{tot} values, more free eigenmodes are available, however the channel condition is poor due to the low available power. Whereas, at high P_{tot} values, despite the good channel condition, less eigenmodes are available. Consequently, the secondary rate reaches a maximum in the mid-range values of P_{tot}. Thus the secondary transmitter should be aware about the optimal power P_{tot} that achieves the maximum rate otherwise the rate will decrease as P_{tot} exceeds this optimal power shown in Figure 2.

Figure 3 shows the effect of multi-antenna diversity on of the SU rate, with $P_{tot} = 20$ dB, considering perfect SIC and for different values of I_{peak}. We notice that the slope of the rate variation is almost linear, and the reaches the upper limit (i.e absence of the PU) above 10 antennas for $I_{peak} = 10$ dB.

In Figure 4, we consider a degenerate SIC scheme with fixed I_{peak}. We notice that the rate presents again a maximum before decreases, in this case, to zero for high SNR. However, within this extreme case, the rate reaches a higher values (up to 3 BPCU) comparing to the FE-PA precoding. Meanwhile, by tolerating a fixed interference threshold, the primary rate presents a constant decrease especially for high SNR. Note that this decrease corresponds to the tolerated interference and, thus, does not affect the QoS of the PU.

In Figure 5, we present the achievable rate region of the PU and the SU, with perfect SIC ($\alpha = 1$) and with degenerate SIC ($\alpha = 0$). We notice, again the maxima of the cognitive rate when the primary rate varies. In the degenerate SIC case, for the same average power, in order to achieve the maximum rate, the primary should not exceed some rates (about 20 BPCU) otherwise the cognitive achievable rate will decrease drastically. However, when this SIC is perfect, the maximum cognitive rate remains almost the same for high primary rate,

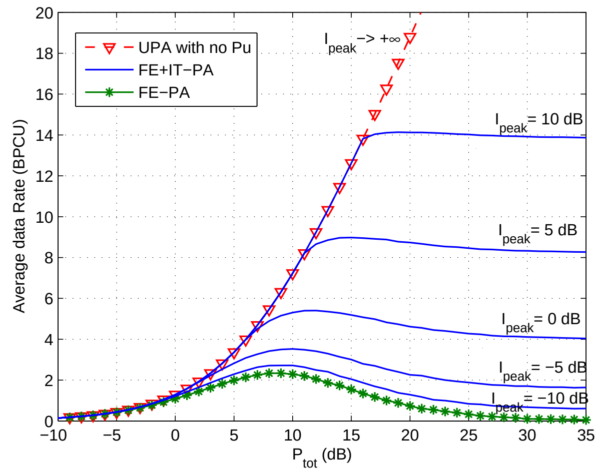

Figure 2. Rate of the CR user with perfect SIC as function of P_{tot} with $N = 4$

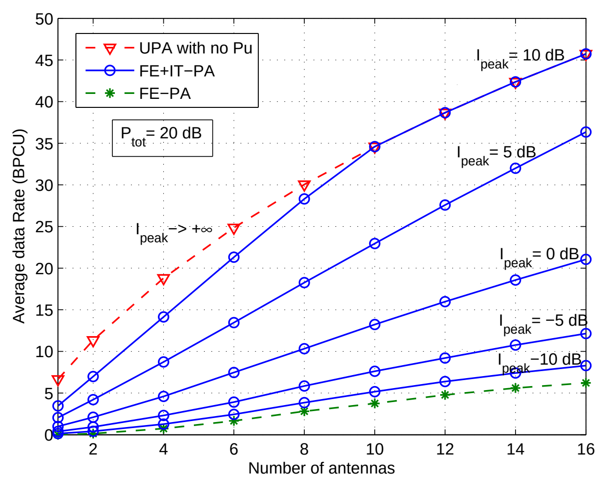

Figure 3. Rate of the CR user as a function of the number of antennas N with P_{tot} = 20dB

which requires an efficient SIC operation at the receiver.

VI. Conclusion

In this paper, we have studied the achievable rate of the secondary cognitive user in a spectrum sharing MIMO uplink communication using a special precoding scheme. The secondary user exploits the unused eigenmodes of the primary user and shares the used ones by respecting both an average power and an interference temperature constraints. We have derived the optimal power that maximizes the secondary user achievable rate for different levels of SIC probabilities of success and we showed that the secondary achievable rate increases significantly when the secondary user, in addition to exploiting the free eigenmodes, shares the used eigenmodes ”responsibly”. The impact of a successful SIC at the common receiver on the secondary rate has also been highlighted. Finally, we presented a protection protocol of primary commu-

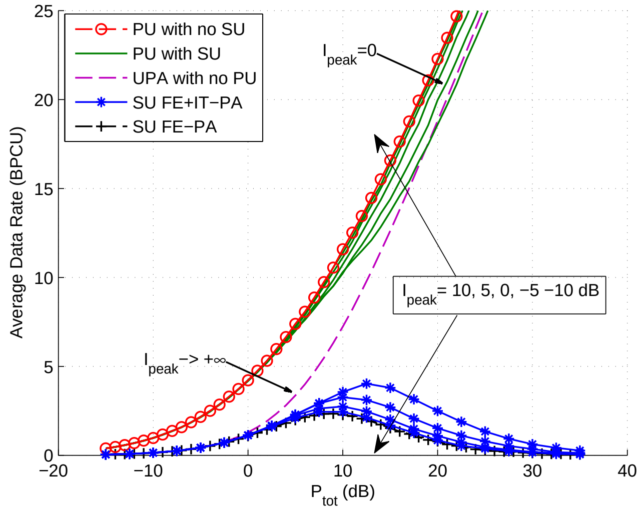

Figure 4. Rate of the SU with degenerate SIC and fixed I_{peak} as function of P_{tot} with N = 4.

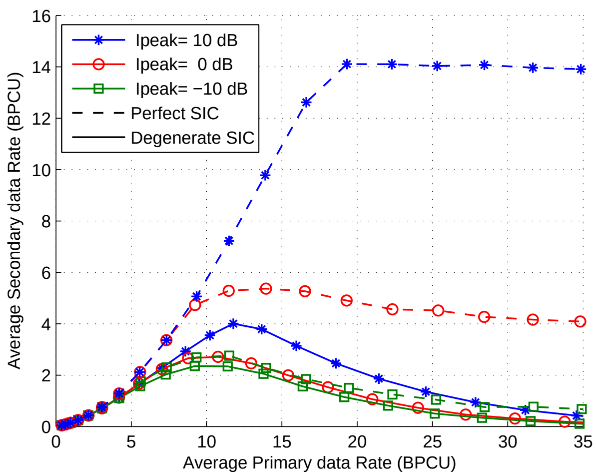

Figure 5. Rate of the SU as function of the rate of the PU.

nication with a fixed interference threshold and we have shown through numerical results that there is a certain threshold of the primary rate that gives the maximum secondary rate with a saturation when the interference threshold is high (grater than 10 dB).

Appendix : Proof of the Theorem

We start by solving two independent subproblems with the objective function (12) and one of the constraints each, similar to the theorem in [16]. Consequently the optimal power is given by the intersection of the two sub-solutions; the first is the usual waterfilling scheme such as (5) and the second is the upper bound of the feasible space which is $I_{peak}(j)$ for $j = 1 \cdots N-n$. Finally, the complete optimal power expression is given by 14. Now, in order to find μ we use the strong duality of our problem since our primal problem is convex [15]. Let $g(\mu)$ to be the dual function given by
$g(\mu) =$

$$\sum_{j=1}^{N-n} \mathbb{E}\left[\log_2\left(1+\min\left\{[\frac{1}{\mu}-1]^+, I_{peak}(j)\right\}\right)\right] + \sum_{j=N-n+1}^{N} \mathbb{E}\left[\log_2\left(1+[\frac{1}{\mu}-1]^+\right)\right]$$
$$+\mu\left(\sum_{j=1}^{N-n}\left(\min\left\{[\frac{1}{\mu}-1]^+, I_{peak}(j)\right\}\right) + \sum_{j=N-n+1}^{N}\left([\frac{1}{\mu}-1]^+\right) - P_{tot}\right).$$

The strong duality imposes that $g(\mu)$ is a concave, and that there exists a unique μ^* that minimizes g and, hence, maximizes the primal function as well. Therefore, given P_{tot}, we compute $\mu^* = argmin_{\mu}\ g(\mu)$ then the optimal power P_s^* and finally we compute the achievable rate C.

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